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New Method of Getting Position of Instrument Station Based on Two Known Points and the Law of Cosines

Totok Sulistyo[*], **Desak Made Ristia Kartika**

Lecturer, Department of Civil Engineering, Politeknik Negeri Balikpapan, Indonesia
Corresponding author: *totok.sulistyo@poltekba.ac.id*

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ABSTRACT

Getting the position of the instrument in starting traverse and staking out surveying can be very helpful for the surveyors. The most common method is the placement of the instrument on the known point, then those surveys are possible to be accomplished. This research is aimed to develop a new method and procedure to get x, y, and z values of the unknown position of the instrument based on two known points and the law of cosines. The method of this research is the implementation of the law of cosines and Euclidean Norm in solving the problem of getting the coordinate of instrument position. The innovation of this procedure has not been used yet in survey practice and has not been accommodated in electronic distance measuring (EDM) based survey instruments such as Total Station. The experiment of measurement to test the procedure is conducted virtually using the total station of SimusurveyX 1.0.7. The total measurement of ten random triangles is 60 times, where each triangle is measured 6 times. The result of measurement is close to the ground truth, and it can be repeatable. The implication of this research is enabling the surveyors to shortcut traverse measurement by locating Total Station in the first unknown point of the traverse.

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1. Introduction

The cosine law can be explored deeply to get the invention of new methods and breakthroughs in determining and predicting the location and position of certain objects. The law of cosine has been proved in determining and predicting position, it has been successfully implemented in the Coordinate Rotation Digital Computer (CORDIC) Processor for location awareness system and robot indoor location technology [1,2]. There are a lot of demands for breakthroughs and new practical methods in determining the position of objects for various kinds of applications in any subject. Such breakthrough is needed urgently in mapping and surveying subjects to facilitate engineers and surveyors in the field to increase their productivity. The breakthrough can be an invention of work instruction that can solve the field problems or procedures which is can be accommodated in the firmware of mapping and surveying instruments or application in ubiquitous devices possible to control the instruments [3]. Some breakthroughs in mapping and surveying such as mobile and computer applications have been developed by software engineers to enhance the productivity of surveyors in the field [4]. The method to stake out or transfer location from the map to the field using are available on the total station software features, and there is an available android app to facilitate such measurement using theodolite or low-cost instruments [5]. Even recently, the method of learning for surveying subjects using a virtual learning environment to enhance student competencies has been developed, and the development computer-based tutorials are being developed [6].

The famous method in starting terrestrial mapping using a Total Station is setting up the instrument in the known position points or known as Benchmark (BM) and setting Back Sight (BS) to the known point, or in other words to determine the location of the target point the instrument must be set on the known point and refers (BS) to the other known points [7]. Based on the operation procedure of Total Station in the mode both of traverse measurement and stake out measurement the surveyor must set up the total station instrument on the known position point and direct the telescope to the known BS point, and the next surveyor can determine Fore Sight (FS) point or locate Stake Out (SO) point. The other method determines roughly the position or coordinates of the point from a couple of known positions by the trilateration method [8,9]. A similar method also has been simulated in MATLAB to get the coordinate of an unknown point using the three-point resection problem [10].

Most wide range products of total stations facilitate users to determine the coordinate of target points based on known backsight reference and the known location of the instrument. The research question in this study is how do we know the current random location of the instrument if there are two available known point positions, and how such kind of method can shortcut the traverse's measurement in certain field conditions. The aim and objective of this research are to develop a new procedure for determining the position of the Total Station in x and y coordinates based on two known position reference points using the law of cosines and how to apply its method in traverse measurement. The implementation of the method of instrument location determination in practical traverse surveying needs accuracy and precision. The accuracy can be defined as the closeness between the agreement or accepted reference value or the truth to a large number of the test results or observations, meanwhile, precision is the closeness of the test results or consistencies of the observation groups and evaluated based on size differences [11–14].

This study proposes a new method of determining position x, y, and z coordinates of random unknown position points based on the distances and angles of two known position points. Such a method has not been accommodated in the method of the stake-out or traverse survey. The Impact of this newly proposed method can solve the field problem when the common procedure of BS cannot be implemented due to the field obstacle. In the future, this new procedure should be accommodated by the latest survey instruments in the built-in application to enhance their performance and will facilitate their users in the field. This new method will enrich the conventional method of the stake-out and traverse survey using a common instruments such as Total Station and Theodolite.

3. Methods

The SimuSurveyX version 1.0.7 is hired in this research activity, in the simulation of measurement experiments, the software is developed by the V-Lab (Visualization Laboratory) Department of Civil Engineering National Taiwan University and can be downloaded freely at https://simusurveyx.caece.net/software/. SimusurveyX provides a realistic effect and learning environment [15]. In running the simulator and the app needs a PC or Laptop with Windows XP; Windows Vista or Windows 7/10 operating system and Minimum system requirement：1.6 GHz CPU、384 Mb RAM、1280 x 720 display. The method of this research consisted of installing the simulator, designing measurement scenarios, calculating the position of the instrument, validating results, determining accuracy and precision, and discussing and drawing conclusions. This method of development is based on the law of cosines which can be explained as follow:

Based on Figure 1, if A, B, and C are the Angles and a, b, and c are the length of sides, therefore it is governed by the following formula [1] which the formula has been successfully implemented in Coordinate Rotation Digital Computer in Location Awareness System.

$$c^2 = a^2 + b^2 - 2ab\ Cos\ C \tag{1}$$

$$b^2 = a^2 + c^2 - 2ac\ Cos\ B \tag{2}$$

$$a^2 = b^2 + c^2 - 2bc\ Cos\ A \tag{3}$$

The length and angle of the triangle can be solved using Formula 1, 2, and 3 [16].

Based on the law of cosines theorem, the Total Station or Instrument Position can be determined precisely through the following procedure of measurement:

1) Setting up the total station in P3 at any location that it has no barrier Line of Sight to P1 as known position and P2 as known position (See Figure 4)
2) Aligning the telescope to P1 as Backsight (BS) set horizontal angle 0° 0’ 0” measure horizontal distance between P3 and P1 note as b (Refers to Figure 4).

3) Rotate telescope to P2, note horizontal angle as C, and measure the distance between P3 and P2 as a,
4) Calculate the length of c or horizontal distance between P1 and P2 using triangular principle or the Pythagorean theorem [2] or Euclidean Norm [17] that can be written as formula (4)

$$c = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \quad (4)$$

5) Based on the law of cosine [1] formula (2), Calculate the angle of B

$$B = arc\cos\frac{a^2+c^2-b^2}{2ac} \quad (5)$$

6) Calculate the azimuth of P1 to P2

$$\alpha = arc\tan\frac{x_2 - x_1}{y_2 - y_1} \quad (6)$$

if $x_2 - x_1 > 0$ and $y_2 - y_1 > 0$ then azimuth ($\alpha_{p1 \rightarrow P2}$) = α (7)

if $x_2 - x_1 > 0$ and $y_2 - y_1 < 0$ then azimuth ($\alpha_{p1 \rightarrow P2}$) = 180 – α (8)

if $x_2 - x_1 < 0$ and $y_2 - y_1 < 0$ then azimuth ($\alpha_{p1 \rightarrow P2}$) = 180 + α (9)

if $x_2 - x_1 < 0$ and $y_2 - y_1 > 0$ then azimuth ($\alpha_{p1 \rightarrow P2}$) = 360 – α (10)

7) Calculate azimuth P2 to P3

$$\alpha_{P2\rightarrow P3} = \alpha_{P1\rightarrow P2} - B + 180 \quad (11)$$

8) Determine the Coordinate of P3 (x_3, y_3, z_3) using this formula

$$x_3 = x_2 + a\sin\alpha_{P2\rightarrow P3} \quad (12)$$

$$y_3 = y_2 + a\cos\alpha_{P2\rightarrow P3} \quad (13)$$

$$z_3 = z_2 - (a\tan(90 - V) + Hi - Hr) \quad (14),$$

i. where V is vertical Angle P1 to P3, Hi is the height of the instrument, and Hr is the height of the reflector or mirror

9) Validate the Coordinate, the result can be validated using the position of Instrument in SimuSurveyX by clicking the escape key and then selecting the result in the screen menu.
10) Determine the error of x and y by comparing mathematical calculations and the instrument's location determined by the simulator.
11) Repeat the experiment of measurement
12) Determine the accuracy and precision of the coordinate of P3 determination using this method.

$$E_{accuracy} = \frac{1}{N}\sum_{i=1}^{N}(R_i - R_{true}) \quad (15)$$

$$Precision = \bar{R} \pm \sqrt{\frac{1}{N-1}\sum_{i=1}^{N}(R_i - \bar{R})^2} \tag{16}$$

13) Assessing accuracy is also can be done using Root Mean Square Error (RMSE) [18] and Mean Average Error using Formula 17 and 18

$$RMSE = \sqrt{\frac{1}{n}\sum_{i=1}^{n}\left(Y_{pi} - Y_{oi}\right)^2} \tag{17}$$

$$MAE = \frac{1}{n}\sum_{i=1}^{n}\left|Y_{pi} - Y_{oi}\right| \tag{18}$$

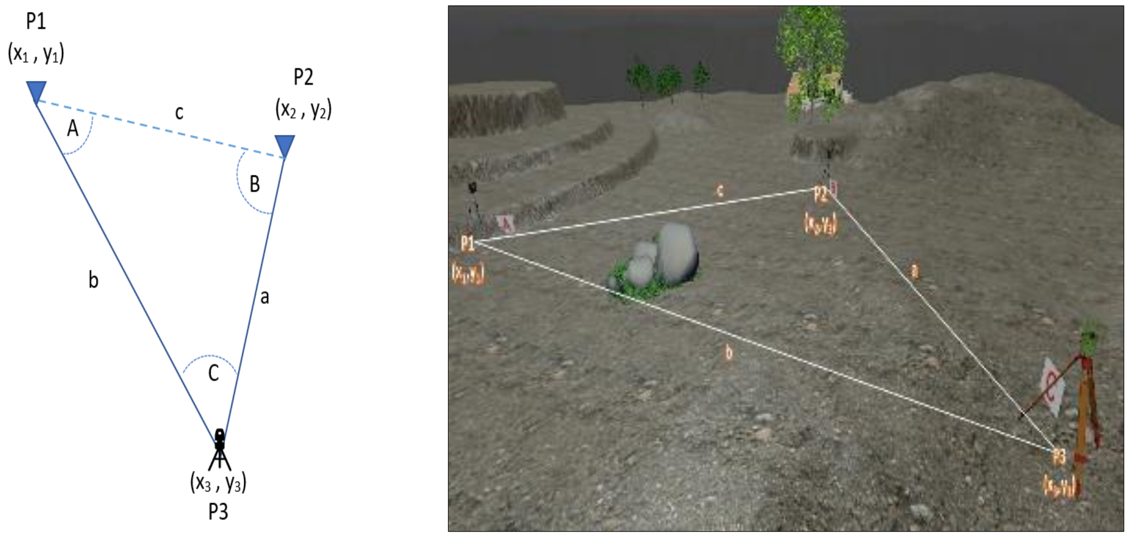


Fig. 1. Two Known Points and The Unknown Point of Total Station Position.

Such calculation simply can be accomplished using a spreadsheet formula, or with little effort, it can be solved by developing certain software by transforming the procedures into algorithms, and algorithms into code. In this research as a tool to get the result from random experiments, the Microsoft Excel formula is developed to solve the problem and compared the result using a simple computer application.

4. Results

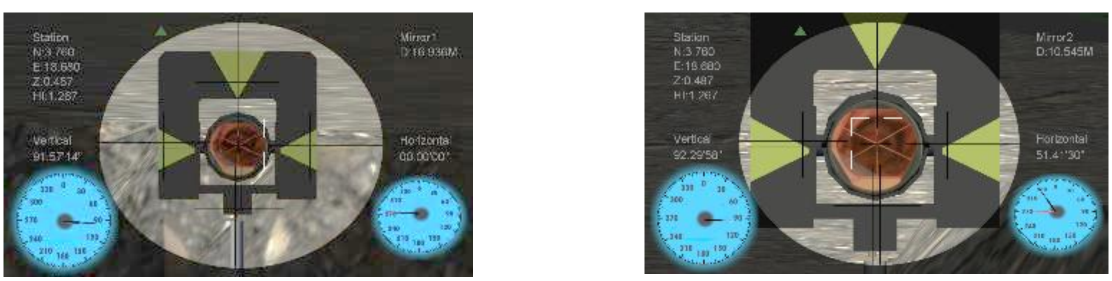
From the instrument setting in the simulator, the position of the mirrors and instrument is depicted in figure 2. The x, y, and z of point A (P1) or mirror 1, point B (P2) or mirror 2 and C (P3) or station

of the instrument is ground truth.

The default error of the virtual instrument axis is set to 0.000 m, as depicted in figure 3, where the x-axis is 0.000 m, the y-axis is 0.000m and the z-axis is 0.000 m.

Fig. 2. Position of Mirrors in Points A or P1 and Point B or P2 and Ground Truth of P3 (Station).

Fig. 3. Error Setting of virtual instrument.



(a) (b)

Fig. 4. (a) Mirror 1, and (b) mirror 2 reading.

In this measurement experiment base on the simulator, the location of rulers and instruments are depicted in figure 4. Where Ruler 1 at point A or P1 and Ruler 2 at point B or P2

x_1 = 6.880 m x_2 =17.640 m

y_1=9.640 m y_2=17.440 m

z_1=30.010 m z_2=30.129 m

meanwhile, figure 4 (b) can be known as following

C = 51° 41' 17" = 51.688°

The distance of P3 (instrument) to P2 (mirror 2)

Instrument height (Hi) = 1.267m

mirror 2 height (Hr) = 1.168 m

a = 10.545 m

the distance of P3 (instrument) to P1 (mirror 1)

mirror 1 height (Hr) = 1.164 m

b = 16.936 m

length of c or the distance of P1 to P2 can be calculated using formula 4

$$c = \sqrt{(17.640 - 6.880)^2 + (17.440 - 9.640)^2} = 13.290 \text{ m}$$

and using the law of cosine formula 5, the angle of B can be calculated

$$B = arc\cos\frac{10.545^2 + 13.290^{\ 2} - 16.936^2}{2\ x\ 10.545\ x\ 13.290}$$

$$= 89.798^{o}$$

The azimuth of P1 to P2 ($\alpha_{P1 \rightarrow P2}$) can be calculated using formula 6 up to formula 10 as follow

$$\alpha = arc\tan\frac{17.640 - 6.880}{17.440 - 9.640}$$

$$= 54.601^{o}$$

Where the condition full fills $x_{1-}\ x_2 > 0\ and\ y_1 - y_2 > 0$, then the Azimuth P1 to P2 is

$\alpha_{P1 \rightarrow P2}$ = N 54.601° E

The next step is, the calculation of the azimuth P2 to P3 ($\alpha_{P2 \rightarrow P3}$) using formula 11

$\alpha_{P1 \rightarrow P2}$ = 54.601 – 89.778 + 180 = N144.263° E

The Coordinate of P3 (x_3, y_3, z_3) using this formula 12 and 13

$$x_3 = 17.640 + 10.545\sin 144.263 = 23.799 \text{ m}$$

$$y_3 = 17.440 + 10.545\cos 144.263 = 8.881 \text{ m}$$

$$z_3 = z_2 - (10.545\tan(90 - 92.499) + 1.267 - 1.168) = 30.490 \text{ m}$$

The above calculation can be solved manually using a calculator, spreadsheet, or computer application. The proposed procedure is transformed into Programming Code and deployed as a simple computer application. The enhancement of functionality and performance of the application is an improvement and is needed to further develop different platforms such as android and IOS [19]. Both calculations using a spreadsheet and developed application have the same result, the development of the application can be adopted by the Total Station manufacturer so that the user or surveyor can run this procedure directly from the instrument. The simple developed computer application to calculate the new procedure is depicted in figure 4.

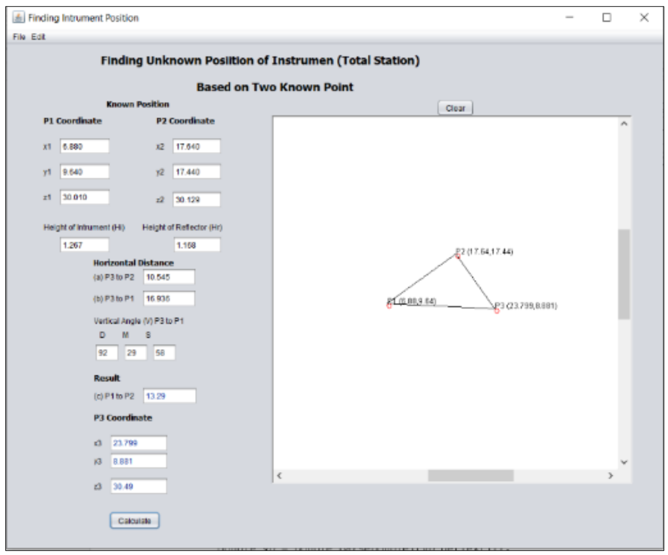


Fig. 4. Computer Program to solve the procedure that is developed using Java.

Table 1
Comparison of x, y, z Measurement, and Ground Truth.

#	P1$_{\text{Ground Truth}}$			P2$_{\text{Ground Truth}}$			P3$_{\text{Measurement}}$			P3$_{\text{Ground Truth}}$			Error (P3$_M$-P3$_{GT}$)		
	x1	y1	z1	x2	y2	z2	x3	y3	z3	x3	y3	z3	x3	y3	z3
1	6.880	9.640	30.010	17.640	17.440	30.129	23.799	8.881	30.490	23.80	8.880	30.487	-0.001	0.001	0.003
	6.880	9.640	30.010	17.640	17.440	30.129	23.802	8.884	30.491	23.80	8.880	30.487	0.002	0.004	0.004
	6.880	9.640	30.010	17.640	17.440	30.129	23.799	8.882	30.490	23.80	8.880	30.487	-0.001	0.002	0.003
	6.880	9.640	30.010	17.640	17.440	30.129	23.802	8.879	30.491	23.80	8.880	30.487	0.002	-0.001	0.004
	6.880	9.640	30.010	17.640	17.440	30.129	23.801	8.881	30.490	23.80	8.880	30.487	0.001	0.001	0.003
	6.880	9.640	30.010	17.640	17.440	30.129	23.802	8.879	30.490	23.80	8.880	30.487	0.002	-0.001	0.003
2	7.257	4.860	30.020	28.333	20.804	30.239	22.252	3.911	30.229	22.25	3.912	30.225	-0.002	-0.001	0.004
	7.257	4.860	30.020	28.333	20.804	30.239	22.255	3.915	30.228	22.25	3.912	30.225	0.001	0.003	0.003
	7.257	4.860	30.020	28.333	20.804	30.239	22.255	3.912	30.229	22.25	3.912	30.225	0.001	0.000	0.004
	7.257	4.860	30.020	28.333	20.804	30.239	22.255	3.914	30.229	22.25	3.912	30.225	0.001	0.002	0.004
	7.257	4.860	30.020	28.333	20.804	30.239	22.256	3.912	30.229	22.25	3.912	30.225	0.002	0.000	0.004
	7.257	4.860	30.020	28.333	20.804	30.239	22.253	3.912	30.230	22.25	3.912	30.225	-0.001	0.000	0.005
3	9.088	12.118	30.020	25.742	20.336	30.122	18.655	2.707	30.840	18.65	2.706	30.836	0.003	0.001	0.004
	9.088	12.118	30.020	25.742	20.336	30.122	18.650	2.705	30.841	18.65	2.706	30.836	-0.002	-0.001	0.005
	9.088	12.118	30.020	25.742	20.336	30.122	18.654	2.706	30.842	18.65	2.706	30.836	0.002	0.000	0.006
	9.088	12.118	30.020	25.742	20.336	30.122	18.652	2.707	30.840	18.65	2.706	30.836	0.000	0.001	0.004

	9.088	12.118	30.020	25.742	20.336	30.122	18.650	2.708	30.842	18.65	2.706	30.836	-0.002	0.002	0.006
	9.088	12.118	30.020	25.742	20.336	30.122	18.653	2.706	30.840	18.65	2.706	30.836	0.001	0.000	0.004
4	15.03	19.900	30.156	27.020	11.825	30.982	9.009	5.388	30.034	9.008	5.390	30.032	0.001	-0.002	0.002
	15.03	19.900	30.156	27.020	11.825	30.982	9.005	5.393	30.034	9.008	5.390	30.032	-0.003	0.003	0.002
	15.03	19.900	30.156	27.020	11.825	30.982	9.007	5.390	30.033	9.008	5.390	30.032	-0.001	0.000	0.001
	15.03	19.900	30.156	27.020	11.825	30.982	9.007	5.393	30.035	9.008	5.390	30.032	-0.001	0.003	0.003
	15.03	19.900	30.156	27.020	11.825	30.982	9.007	5.392	30.032	9.008	5.390	30.032	-0.001	0.002	0.000
	15.03	19.900	30.156	27.020	11.825	30.982	9.006	5.392	30.034	9.008	5.390	30.032	-0.002	0.002	0.002
5	27.74	7.502	30.749	2.283	8.536	30.502	15.181	22.838	31.010	15.18	22.839	31.004	0.000	-0.001	0.006
	27.74	7.502	30.749	2.283	8.536	30.502	15.182	22.836	31.010	15.18	22.839	31.004	0.001	-0.003	0.006
	27.74	7.502	30.749	2.283	8.536	30.502	15.180	22.836	31.009	15.18	22.839	31.004	-0.001	-0.003	0.005
	27.74	7.502	30.749	2.283	8.536	30.502	15.179	22.837	31.007	15.18	22.839	31.004	-0.002	-0.002	0.003
	27.74	7.502	30.749	2.283	8.536	30.502	15.183	22.838	31.009	15.18	22.839	31.004	0.002	-0.001	0.005
	27.74	7.502	30.749	2.283	8.536	30.502	15.180	22.839	31.008	15.18	22.839	31.004	-0.001	0.000	0.004
6	23.10	5.826	30.199	6.935	15.399	30.449	25.941	21.009	30.164	25.94	21.009	30.159	0.001	0.000	0.005
	23.10	5.826	30.199	6.935	15.399	30.449	25.938	21.007	30.164	25.94	21.009	30.159	-0.002	-0.002	0.005
	23.10	5.826	30.199	6.935	15.399	30.449	25.941	21.010	30.161	25.94	21.009	30.159	0.001	0.001	0.002
	23.10	5.826	30.199	6.935	15.399	30.449	25.938	21.009	30.162	25.94	21.009	30.159	-0.002	0.000	0.003
	23.10	5.826	30.199	6.935	15.399	30.449	25.941	21.007	30.163	25.94	21.009	30.159	0.001	-0.002	0.004
	23.10	5.826	30.199	6.935	15.399	30.449	25.937	21.009	30.164	25.94	21.009	30.159	-0.003	0.000	0.005
7	5.970	8.023	30.010	13.940	19.633	30.116	23.041	7.632	30.410	23.04	7.636	30.406	-0.001	-0.004	0.004
	5.970	8.023	30.010	13.940	19.633	30.116	23.044	7.636	30.409	23.04	7.636	30.406	0.002	0.000	0.003
	5.970	8.023	30.010	13.940	19.633	30.116	23.042	7.636	30.408	23.04	7.636	30.406	0.000	0.000	0.002
	5.970	8.023	30.010	13.940	19.633	30.116	23.041	7.632	30.409	23.04	7.636	30.406	-0.001	-0.004	0.003
	5.970	8.023	30.010	13.940	19.633	30.116	23.044	7.636	30.409	23.04	7.636	30.406	0.002	0.000	0.003
	5.970	8.023	30.010	13.940	19.633	30.116	23.040	7.632	30.410	23.04	7.636	30.406	-0.002	-0.004	0.004
8	5.794	11.524	30.502	27.714	19.390	30.142	19.121	4.279	30.861	19.12	4.280	30.855	0.001	-0.001	0.006
	5.794	11.524	30.502	27.714	19.390	30.142	19.120	4.281	30.861	19.12	4.280	30.855	0.000	0.001	0.006
	5.794	11.524	30.502	27.714	19.390	30.142	19.120	4.280	30.860	19.12	4.280	30.855	0.000	0.000	0.005
	5.794	11.524	30.502	27.714	19.390	30.142	19.118	4.280	30.860	19.12	4.280	30.855	-0.002	0.000	0.005
	5.794	11.524	30.502	27.714	19.390	30.142	19.122	4.279	30.860	19.12	4.280	30.855	0.002	-0.001	0.005
	5.794	11.524	30.502	27.714	19.390	30.142	19.120	4.279	30.860	19.12	4.280	30.855	0.000	-0.001	0.005
9	11.35	20.054	30.050	24.620	11.377	30.626	9.803	2.089	30.052	9.802	2.088	30.050	0.001	0.001	0.002
	11.35	20.054	30.050	24.620	11.377	30.626	9.797	2.092	30.052	9.802	2.088	30.050	-0.005	0.004	0.002
	11.35	20.054	30.050	24.620	11.377	30.626	9.805	2.087	30.052	9.802	2.088	30.050	0.003	-0.001	0.002
	11.35	20.054	30.050	24.620	11.377	30.626	9.802	2.086	30.052	9.802	2.088	30.050	0.000	-0.002	0.002
	11.35	20.054	30.050	24.620	11.377	30.626	9.801	2.087	30.054	9.802	2.088	30.050	-0.001	-0.001	0.004
	11.35	20.054	30.050	24.620	11.377	30.626	9.804	2.087	30.053	9.802	2.088	30.050	0.002	-0.001	0.003
10	8.091	17.286	30.010	29.721	3.851	30.289	14.114	4.946	30.245	14.11	4.943	30.244	-0.002	0.003	0.001
	8.091	17.286	30.010	29.721	3.851	30.289	14.116	4.945	30.244	14.11	4.943	30.244	0.000	0.002	0.000
	8.091	17.286	30.010	29.721	3.851	30.289	14.114	4.943	30.245	14.11	4.943	30.244	-0.002	0.000	0.001
	8.091	17.286	30.010	29.721	3.851	30.289	14.116	4.941	30.245	14.11	4.943	30.244	0.000	-0.002	0.001
	8.091	17.286	30.010	29.721	3.851	30.289	14.116	4.942	30.245	14.11	4.943	30.244	0.000	-0.001	0.001
	8.091	17.286	30.010	29.721	3.851	30.289	14.117	4.944	30.245	14.11	4.943	30.244	0.001	0.001	0.001

The list of java language code for button action event functions is as follows:

```
private void btCalculActionPerformed(java.awt.event.ActionEvent evt) {
        // TODO add your handling code here:
        double x1 = Double.parseDouble(tx1.getText());
        double y1 = Double.parseDouble(ty1.getText());
        double z1 = Double.parseDouble(tz1.getText());
        double x2 = Double.parseDouble(tx2.getText());
        double y2 = Double.parseDouble(ty2.getText());
        double z2 = Double.parseDouble(tz2.getText());
        double hr = Double.parseDouble(thr.getText());
        double hi = Double.parseDouble(thi.getText());
        double a = Double.parseDouble(ta.getText());
        double b = Double.parseDouble(tb.getText());
        double vD = Double.parseDouble(tvD.getText());
        double vM = Double.parseDouble(tvM.getText());
        double vS = Double.parseDouble(tvS.getText());
        double V = vD + (vM/60)+ (vS/3600);
        double azimuth12=0;
        double azimuth23=0;
        double c = pow((pow(x2 - x1,2))+(pow(y2 -y1,2)),0.5);
        double deg = (pow(a,2)+pow(c,2)-pow(b,2))/(2*a*c);
        double B = Math.toDegrees(Math.acos(deg));
        double az1 = (x2 - x1)/(y2 - y1);
        double az=Math.toDegrees(Math.atan(az1));
        //double azimuth12 = az;
        double x3 = 0;
        double y3 = 0;
        double z3;
        if ((x2 - x1)>0 && (y2 - y1)>0){
           azimuth12 = az;
           azimuth23 = azimuth12 - B +180;
           x3 = x2 + a*(Math.sin(Math.toRadians(azimuth23)));
```

```
            y3 = y2 + a*(Math.cos(Math.toRadians(azimuth23)));
        }
        else if ((x2 - x1)>0 && (y2 - y1)<0){
            azimuth12 = 180 - az;
            azimuth23 = azimuth12 - B +180;
            x3 = x2 + a*(Math.sin(Math.toRadians(azimuth23)));
            y3 = y2 + a*(Math.cos(Math.toRadians(azimuth23)));
        }else if ((x2 - x1)<0 && (y2 - y1)<0){
            azimuth12 = 180 + az;
            azimuth23 = azimuth12 - B + 180;
            x3 = x2 + a*(Math.sin(Math.toRadians(azimuth23)));
            y3 = y2 + a*(Math.cos(Math.toRadians(azimuth23)));
        }else if((x2 - x1)<0 && (y2 - y1)>0){
            azimuth12 = 360 - az;
            azimuth23 = azimuth12 - B + 180;
            x3 = x2 + a*(Math.sin(Math.toRadians(azimuth23)));
            y3 = y2 + a*(Math.cos(Math.toRadians(azimuth23)));
        }
        z3 = z2 - (a*Math.tan(Math.toRadians(90 - V))+(hi - hr));
        DecimalFormat df = new DecimalFormat("###.###");
        tc.setText(String.valueOf(df.format(c)));
        tx3.setText(String.valueOf(df.format(x3)));
        ty3.setText(String.valueOf(df.format(y3)));
        tz3.setText(String.valueOf(df.format(z3)));
    }
```

And then check the Error of measurement and calculation by comparing x3, y3, and z3 of measurement to the ground truth of P3 to get the error value. It is known station P3 has x_3 = 23.800 m, y_3 = 8.880 m, and z_3 = 30.487 m. The error of x_3 measurement is 0.001 m, the error of y_3 is 0.001 m, and the error of z_3 is 0.003 m. In this research 10 triangles with two different known locations of points and random total station positions were set, each triangle is measured 6 times to determine accuracy and precision. Table 1 is the results of the measurement, then calculated the difference in x or dx is the result of x_3 -x_2, dy, and dz, mean, standard deviation, and relative standard deviation as listed in table 2.

The Accuracy and precision in the experiment of measurement 1:

$$Average\ Accuracy_x = 0.001\ m$$
$$Average\ Accuracy_y = 0.001\ m$$
$$Average\ Accuracy_z = 0.003\ m$$

$SD_x = 0.001\ m$ therefore, precisions of dx measurement is 6.161 ± 0.001 m
$SD_y = 0.001\ m$ therefore, precisions of dy measurement is 6.161 ± 0.001 m
$SD_z = 0.001\ m$ therefore, precisions of dz measurement is 6.161 ± 0.001 m

Tabel 2
Value of Average Errors and % Error of x y z compared to the true differential distance value of (P3 – P2).

Measurement (i)		Measurement (meter)			Average $P3_M - P3_{GT}$		
		$dx_{p2->p3}$	$dy_{p2->p3}$	$dz_{p2->p3}$	x	y	z
1	Average	6.161	-8.559	0.361	0.001	0.001	0.003
	SD	0.001	0.002	0.000			
2	Average	-6.079	-16.891	-0.010	0.000	0.001	0.004
	SD	0.001	0.001	0.001			
3	Average	-7.089	-17.629	0.719	0.001	0.001	0.005
	SD	0.002	0.001	0.001			
4	Average	-18.013	-6.434	-0.948	-0.001	0.001	0.002
	SD	0.001	0.002	0.001			
5	Average	12.898	14.301	0.507	0.000	-0.002	0.005
	SD	0.001	0.001	0.001			
6	Average	19.004	5.610	-0.286	-0.001	0.000	0.004
	SD	0.002	0.001	0.001			
7	Average	9.102	-11.999	0.293	0.000	-0.002	0.003
	SD	0.002	0.002	0.001			
8	Average	-8.594	-15.110	0.718	0.000	0.000	0.005
	SD	0.001	0.001	0.000			
9	Average	-14.818	-9.289	-0.573	0.000	0.000	0.003
	SD	0.002	0.002	0.001			
10	Average	-15.605	1.092	-0.044	0.000	0.000	0.001
	SD	0.001	0.002	0.000			
Grand Average					**0.000**	**0.001**	**0.003**

The accuracy and precision of the measurement are graphically depicted in figure 5 where the figure 5 (a) x, y scatter plot shows dots, close to each other in the center of x = 0.000 and y =0.000, while 2 other scatter plots (b) and (c) where vertical axis represent z, dots close each other but the center in z = ±0.004, (d) in three dimensional scatter view showing several dots occupy on the above of spherical graphs (radius=0.005m, and center x=0.000m, y=0.000m, z=0.000m) means inaccurate, Meanwhile (e) all dots close each other and covered by sphere graph which depicts the measurement precision and the center of the assemblage of dots are (0.000, 0.000, 0.004) means z measurement less accurate than x and y.

The accuracy of measurement using the newly proposed method has proof that horizontally has high accuracy ± 1 mm and the result of measurement repeatable with a maximum standard deviation of 2 mm. The less accurate issue in z value is a common problem for the instrument such as Total Station, the vertical error depends on the vertical angle, the greater the vertical angle then the result a higher error in z measurement [20], the higher accuracy is needed the height difference measurement can be determined using auto-level [21].

In the common procedure of the SO and traverse measurement where the instrument of Total Station must occupy the known position point and reading mirror or prism to the other known position point as reference or BS, then rotate the telescope to the unknown position point as target point that will be determined its position. In BS measurement the Total Station needs the free line of sight (LOS) so that the EDM transmits the beam to the prism, in which the prism returns the beam to receiving optic and will be converted to an electrical signal [7]. When field obstacles such as a tree, or building are found, that causes a barrier between the first benchmark point and the second benchmark known point, so the instrument and prism have no LOS. In such a field problem, the new proposed procedure can be applied as a solution. The implementation of the new procedure in surveying is depicted in Fig. 6.

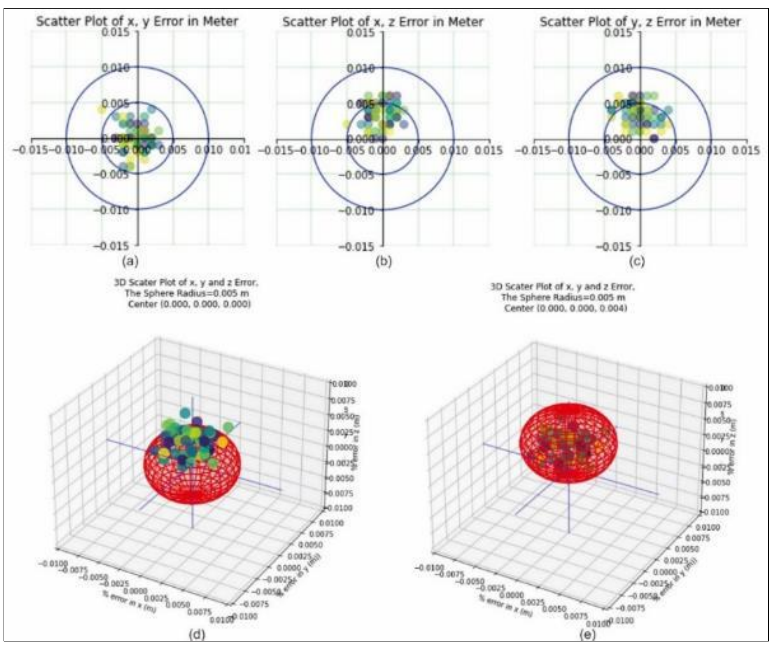
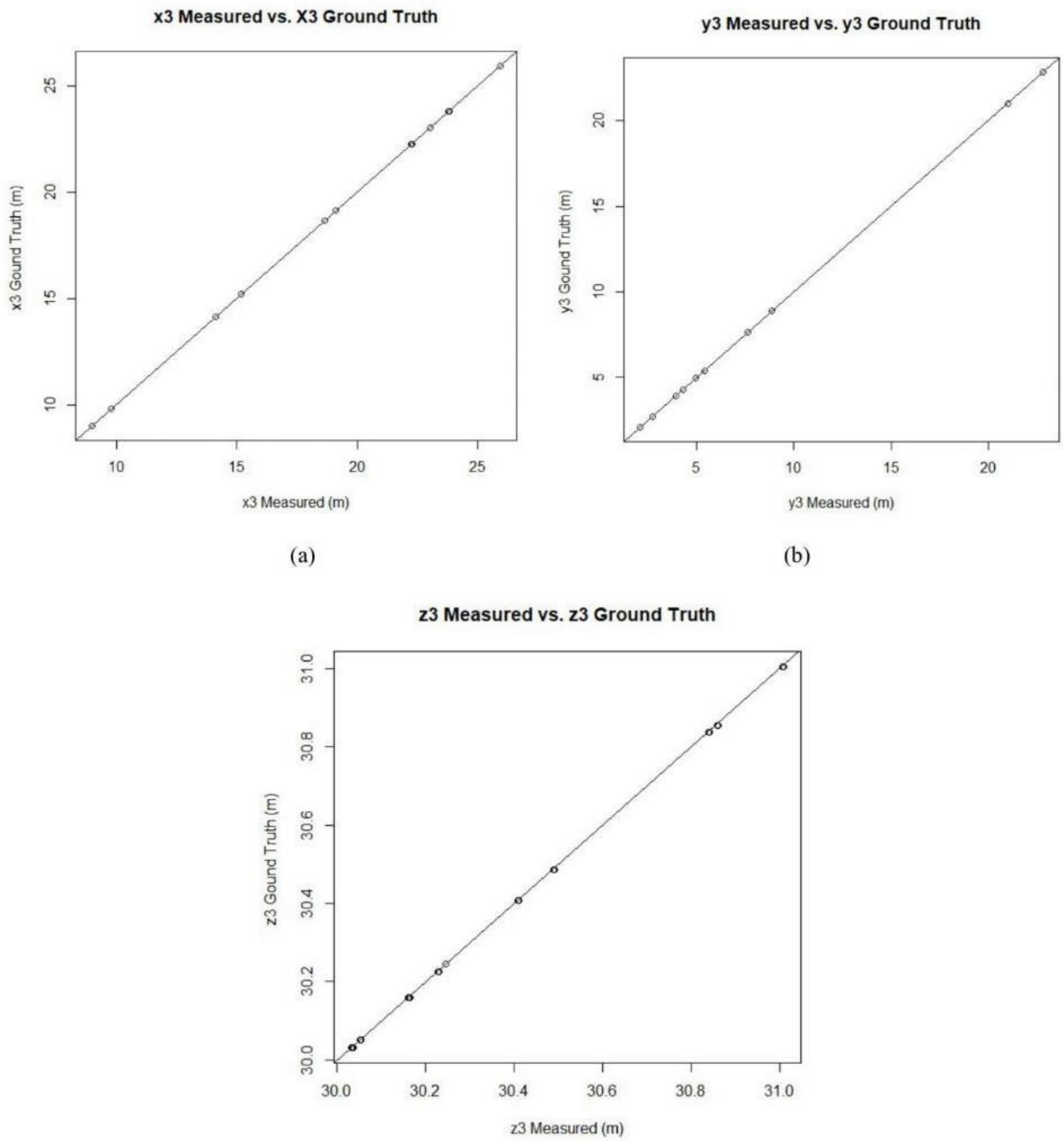


Fig. 5. The 3D scatter Plot of x, y, and z error in meter (a) shows horizontally x and y precise and accurate, meanwhile (b, c, d) vertically z less accurate and but (e) relatively precise. The sphere has a radius of 0.005 meters.

The accuracy and precision also can be assessed from the scatter plot measured location of P3 (x3, y3, z3) versus the Ground Truth of P3 (x3, y3, z3) as illustrated in Figure 6. It shows the scatter dots relatively fit the line of accuracy and the position of dots in each group of observations shows the result is replicable which is prove the precision of the new method in general. The value of RMSE and MAE from measured and Ground Truth is calculated based on the formulas 17 and 18 using the R Studio function as listed in Table 3. This shows the most accurate measurement is on the y-axis while the value of the position on the z-axis is the most inaccurate, although in general, the measured value has high accuracy which is proven by the small value of RMSE and MAE.



(c)

Fig. 6. Plots of Measured value versus Ground Truth.

Table 3

Value of RMSE and MAE of Measured x3, y3, and z3 result of the function (rmse) and function (mae) in R Studio.

Position Coordinate	RMSE	MAE
x_3	0.001678293	0.001383333
y3	0.001839384	0.001383333
z_3	0.003796929	0.00345

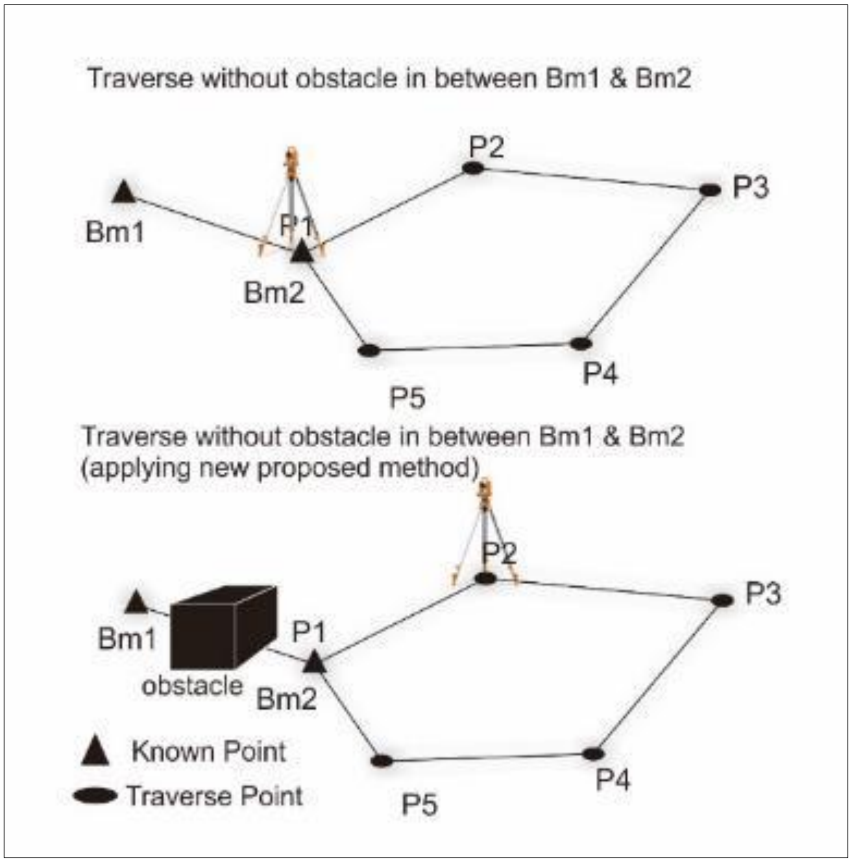


Fig. 7. A common method to measure traverse measurement [7] in the upper, and the lower is the proposed new method that can be used when there is an obstacle between two known points.

5. Discussion

The result of a virtual measurement can be calculated in finding the predicted location, meanwhile, the simulator has accurate information of ground truth where the instrument is set in the field. Such a feature of SimusurveyX has better information than the real grid field, the simulator will detect every movement of the instrument with millimeters accuracy, and such detection will be reflected in the northing, easting, and elevation coordinates of the instrument station. Therefore, those features can be utilized to prove the proposed new method in the self-defining location where it will be a little bit more difficult when must be proven using the real instrument in the real field. The weakness of this newly proposed method is inaccuracy in vertical or elevation measurement, it could be caused by the inaccuracy in determining the height of the reflector, as the reflector height is not automatically informed by the simulator, but it must be determined using virtual ruler, the virtual ruler is place side by side to the reflector, and then reflector height is defined based on the scale of the virtual ruler. The finding of the new method can be translated into the algorithm and the algorithm can be translated into the certain application. In testing the proposed new method, simple software based on java has been developed. The software development in the self-positioning system using the newly proposed method has a great opportunity to enrich the standard built-in application software in Total Station because there is no availability of built-in software features of this newly proposed method in the recent product of Total Station. There is a great opportunity for users of Total Station with the Android Operating System, the user is easier to develop their app to enhance the functionality of the instrument. Users can create android apps in a situation where they find problems or obstacles in starting the traverse procedure and data processing. In replacing standard spreadsheets and desktop applications those have more hardship to use directly in the field.

This proposed new procedure is also can be implemented when surveyors use another instrument such as theodolite in the field. The strengths of the new proposed procedure gives higher accuracy and precision horizontally it also will reduce time in setting the instrument in the field, as the instrument does not need to be set right on the BMs or pegs. And the weakness of the new procedure is the vertical position has lesser accuracy rather than the vertical position. So this new method will be matched when the surveyor just needs x and y coordinates as depicted in Fig. 5. According to the strength and weaknesses of the proposed new procedure, it is suitable for planimetric and cadastral measurements. When users need higher accuracy in height differences or elevation, they can combine the measurement using Total Station for x and y coordinates and auto-level for z coordinates. In the future, this procedure can be used to develop a positioning system instrument based on distance and angle from two known positions. Its possible application is not merely just in the surveying engineering field, but it has a chance to be applied in the navigation system and other problems in a self-positioning system.

6. Conclusion

Both traverse and stake-out surveys sometimes find new obstacles in conducting setting the instrument. The common obstacle there is a barrier between two known points so the instrument does not have a line of sight to the backsight point. Such conditions need a new procedure. The new proposed procedure in determining position based on two known position points and the law of cosine has proven to determine the coordinate of instrument position, the procedure is tested using virtual measurement data and it has high accuracy in x and y between 0 to 2 mm, meanwhile vertically z has lower accuracy between 1 to 5 mm. This new procedure can be used to overcome the problem in stake out and polygon measurement when the surveyor finds an obstacle between two known points and the old procedure cannot be applied in the field. The new procedure starts by setting Total Station or Theodolite in a random unknown position as long as the instrument can read prisms or rulers in two known points. Then the location of the instrument could be determined accurately and precisely.

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Conflicts of interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

Authors contribution statement

TS: Conceptualization; TS, DMRK: Data curation; TS: Formal analysis; TS: Investigation; TS: Methodology; TS: Project administration; TS: Resources; TS: Software; TS: Supervision; DMRK: Validation; TS: Visualization; TS: Roles/Writing – original draft; TS, DMRK: Writing – review & editing.

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