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Gaussian Process Regression (GPR) for Auto-Estimation of Resilient Modulus of Stabilized Base Materials

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ABSTRACT

The resilient modulus of different pavement materials is one of the most important parameters for the pavement design using the mechanistic-empirical (M-E) method. The resilient modulus is generally determined by a triaxial test, which is expensive and time-consuming and requires special laboratory facilities. This study aims to develop a model based on the Gaussian Process Regression (GPR) to predict the resilient modulus of stabilized base material with different additives under wetting-drying cycles. For this purpose, a laboratory dataset containing 704 records have been used. The input parameters were considered as the wetting-drying cycles, free lime to silica ratio, Alumina and iron oxide compounds in the additives, maximum dry density to optimum moisture content ratio, deviator stress, and confining stress. The results indicate high accuracy of the GPR method with a regression coefficient of 0.997 and 0.986 respectively for train and test data and 0.995 for all datasets. Comparing the developed model based on the GPR method with the developed models in the literature based on the artificial neural network methods and the support vector machines shows higher accuracy of the GPR method.

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1. Introduction

The resilient modulus (M_r) is measured to determine the material stiffness values within different stress levels and describe the stress-strain nonlinear behavior of soil under repeated loading. This parameter is one of the important key parameters for pavement analysis and design. In 1986, the AASHTO pavement design guide recommended using the M_r to describe the stiffness of subgrade soil [1]. The M_r has been used as one of the fundamental characteristics to describe materials in structural design of flexible pavement since the publication of the AASHTO guideline in 1986 [2]. Moreover, the resilient modulus is also used to describe soil characteristics and aggregate materials in the mechanistic-empirical pavement design methods (e.g MEPDG method) [3].

The resilient modulus of soil and the aggregate materials is defined as the applied deviator stress to recoverable axial strain ratio under dynamic loading as follows [4]:

$$M_r = \frac{\sigma_d}{\varepsilon_r} \quad (1)$$

Where σ_d is the deviator stress and ε_r is the recoverable axial strain. In the uniaxial compression test without confined stress, σ_d equals to the axial stress.

The resilient modulus is directly determined in the laboratory by dynamic triaxial test, resonance column test, torsional shear test, and gyratory methods [5–7] or indirectly through correlation with the results of other standardized tests or backcalculation [8]. The most common method to determine the resilient modulus of soil and the aggregate materials in the laboratory is the dynamic triaxial test under the effect of different confining and deviator stresses. The dynamic triaxial test is time-consuming and expensive therefore, it is useful to present new methods and techniques to obtain an accurate estimation of the resilient modulus. Different statistical and computational intelligence models have been developed to predict the resilient modulus of fine-grained and coarse-grained soil so far, including the models developed based on the artificial neural network [9–13], support vector machines [14–16], adaptive neuro-fuzzy inference system [17] as well as hybrid models [18].

The stabilized road materials should be sufficiently durable under the effect of Wetting and Drying (W-D) as well as Freezing and Thawing (F-T) cycles [19]. According to the mechanistic-empirical pavement design guidelines, among other factors, the W-D and F-T cycles are important factors which cause damage and deterioration of base and subbase materials and likely lead to premature failure of the pavement [20]. Several researches have been carried out concerning the evaluation of the effect of W-D and F-T cycles on the mechanical and physical properties of the stabilized base layer [21–26]. In this regard, George and Davison (1963) carried out an experimental study to evaluate the durability of stabilized fine-grained soils under F-T cycles. The results of their study showed that applying ten F-T cycles is sufficiently destructive for the stabilized base layer [27]. Nonan and Hamfri (1990) evaluated the effect of the F-T cycles on the resilient modulus of the limestone aggregates stabilized with cement. Their results indicated the importance of the F-T cycles over the W-D cycles at low deviator stresses. Moreover, they found out that the resilient modulus of the stabilized aggregates decreases significantly during the primary stages of the F-T cycles [21]. To evaluate the stabilized soils in Oklahoma, Miller et al. (2000) subjected stabilized samples to W-D cycles according to the ASTM D559 test method. In this study, the unconfined compressive strength (UCS) test was employed to evaluate durability. Results showed that the UCS increases by the increase in the W-D cycles [28]. Khoury and Zaman (2002) evaluated the effect of the W-D cycles on low quality aggregates stabilized with class C fly ash. Aggregates with low quality are referred to as the final acceptable limit to be used in the base layer in Oklahoma. In their study, the resilient modulus, the UCS test, and the elastic modulus were used to evaluate the effect of W-D cycles. The results showed that the resilient modulus values of the cured specimens in 28 days under 30 W-D cycles are approximately 5% lower than the resilient modulus of similar specimens without exposure to the W-D cycles [29]. By evaluating the effect of F-T cycles on the flexural characteristics of the aggregates stabilized with 10% of class C fly ash, Khoury and Zaman (2000) found out that the resilient modulus and rupture modulus decrease as the F-T cycles increase. Based on the results, they noted that the effect of F-T cycles on these two mechanical properties is a function of curing time and the number of F-T cycles [30]. They also explored the effect of the W-D cycles on the resilient modulus of the aggregate materials stabilized with different stabilizers in 2007 [31]. The results showed that the changes in the resilient modulus under W-D cycles can be determined by the rate of the chemical reaction. It was also stated that changing the values of the resilient modulus can be expressed more properly by lime content, SFA (SiO_2+Al_2O_3+Fe_2O_3) content, the optimum moisture content, and the maximum dry density. These researchers also proposed a regression model to predict the effect of these parameters as well as W-D cycles on the resilient modulus changes [31]. Maloof et al (2012) used the support vector machine (SVM) to predict the resilient modulus of the aggregate stabilized materials subjected to W-D cycles. They showed that the support vector machine results in more accurate modeling in comparison with the Least Square (LS) method [20]. Ghanizadeh and Rahravan (2016) used the artificial neural network (ANN) to predict the resilient modulus of the stabilized base materials under W-D cycles and compared their results with Maloof et al (2012) study. Results of this study confirms higher accuracy of ANN in comparison with the SVM method [9]. Kaloop et al (2019) developed some models to predict the resilient modulus of the stabilized base materials by combining the particle swarm optimization algorithm with the artificial neural network (ANN-PSO) and the extreme learning machine (PSO-ELM) and concluded that the PSO-ELM method has higher accuracy compared to other methods [32].

This study aims to present a model based on the Gaussian process regression (GPR) to predict the resilient modulus of stabilized base materials subjected to W-D cycles. The developed model is compared to the models developed by other researchers. The importance of each of the parameters to predict the resilient modulus of the stabilized base materials is considered by the sensitivity analysis. Also, the effect of each of the input parameters on the resilient modulus is evaluated by parametric analysis.

2. Dataset

In this research, the dataset was adopted from Maloof et al. (2012) study [20]. In this dataset, four types of aggregate including Meridian, Richard Sput, Sawyer, and Rhyolite were stabilized by different additives and tested under different W-D cycles to determine the resilient modulus. The Meridian is a limestone material with 97% calcium carbonate ($CaCO_3$) and Richard Spur is a limestone material with 87% calcium carbonate. Also, the Sawyer aggregate is a type of sandstone having a SiO_2 amount of about 94% [20].

The stabilizer agents in Maloof et al. study included Cement Kiln Dust (CKD), Class C Fly ash (CFA), and Fluidized Bed Ash (FBA). The specimens were made near the optimum moisture content and maximum dry density and were cured within 28 days with an approximate temperature of 21^{o}C and humidity percentage was about 90%. Then the specimens were exposed to 0, 8, 16, and 30 W-D cycles, and the resilient modulus of the stabilized materials were obtained to evaluate the performance of the stabilized materials subjected to W-D cycles. The resilient modulus test was carried out by applying a haversine dynamic load with loading time of 0.1s and rest period of 0.9s [20]. Finally, a dataset containing 704 records, five input variables and one output variable was stablished [20].

Previous studies show that the resilient modulus of the stabilized aggregate base is a function of the Wetting-Drying Cycles (WDC), Calcareous/Siliceous Fly Ash Ratio (CSFAR), the dry Density to Moisture content Ratio (DMR), confining stress (σ_3) and deviator stress (σ_d) [31]. Therefore, in the current study these input variables are used to develop the GPR model.

The statistical characteristics along with the frequency and the cumulative frequency for this dataset is shown in Figure 1.

3. Modeling by the Gaussian process regression (GPR)

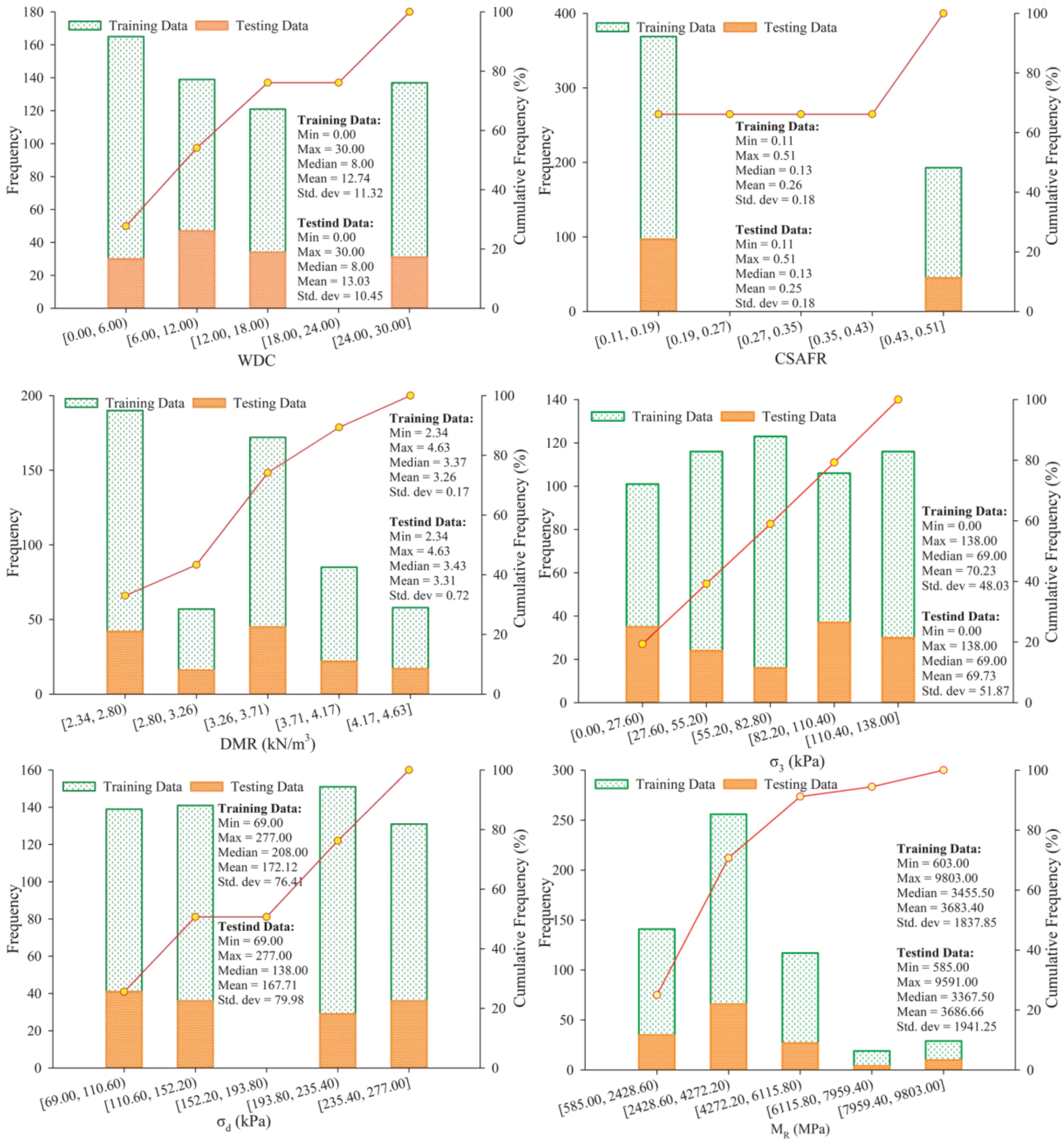
3.1. Gaussian process regression (GPR)

The Gaussian process regression is a probabilistic non-parametric learning method which is widely used for regression and classification problems [33]. This method has attracted the attention of many researchers in various scientific fields [34,35]. GPR is highly efficient for modeling the nonlinear data due to kernel functions. Moreover, the main merit of GPR is providing a reliable response for the input data [36].

Assume that in a training set of $D = \{(x_i, y_i) | i = 1,...,n\}$, $X \in R^{D \times n}$ is the input data points (design matrix) and $y \in R^n$ is the desired output vector. The main assumption of GPR is that the output can be calculated as follows [37,38]:

$$y = f(x) + \varepsilon \tag{2}$$

Where $\varepsilon \sim N(0, \sigma_n^2) \in R$ is the equal noise variance for all x_i samples.



WDC: the number of Wetting-Drying Cycles

CSAFR: Calcareous to Siliceous Fly Ash ratio (silica, Alumina, and iron oxide compounds in cementitious materials)

DMR: Dry Density to Moisture content Ratio

σ3: Restrictive stress (kPa)

σd: Deviant stress (kPa)

MR: resilient modulus (MPa)

Fig. 1. The statistical characteristics, frequency and cumulative frequency of inputs and output variables.

In the GPR method, n observation in $y = \{y_1, \ldots, y_n\}$ vector is considered as a single point instance of the Gaussian multivariate distribution. In addition, it can be assumed that this Gaussian distribution has a mean of zero. The covariance function $k(x, x')$ determines the relationship of one observation to another. The square of the exponential covariance function is commonly used to approximate functions using the GPR method, which is as follows [34,38]:

$$k(x,x') = \sigma_f^2 \times \exp\left(-\frac{(x-x')^2}{2l^2}\right) + \sigma_n^2 \delta(x,x') \tag{3}$$

Where the maximum allowable covariance is defined as σ_f^2. It is noteworthy that $k(x,x')$ is equal to the maximum allowable covariance only when x and x' are so close to each other, therefore, $f(x)$ is approximately equal to $f(x')$. Besides, l shows the length of the kernel function. Moreover, $\delta(x,x')$ is Kronecker delta function which is defined as follows:

$\delta_{ij} = 1$ if $i = j$ and $\delta_{ij} = 0$ if $i \neq j$.

Concerning the training dataset, the final objective of the learning process is to predict y_* the output value for a new input pattern. To achieve this, it is essential to develop three covariance matrices as follows:

According to the training data set, the ultimate goal of the learning process is to predict the output value of y_* for a new input pattern. To achieve this goal, it is necessary to create three covariance matrices as follows:

$$\begin{aligned} K &= \begin{bmatrix} k(x_1,x_1) & k(x_1,x_2) & \ldots & k(x_1,x_n) \\ k(x_2,x_1) & k(x_2,x_2) & \ldots & k(x_2,x_n) \\ \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots \\ k(x_n,x_1) & k(x_n,x_2) & \ldots & k(x_n,x_n) \end{bmatrix} \\ K_* &= \begin{bmatrix} k(x_*,x_1) & k(x_*,x_2) & \ldots & k(x_*,x_n) \end{bmatrix} \\ K_{**} &= k(x_*,x_*) \end{aligned} \tag{4}$$

Regarding the assumption that the data are taken from a Gaussian multivariable distribution:

$$\begin{bmatrix} y \\ y_* \end{bmatrix} \sim N\left(0, \begin{bmatrix} K & K_*^T \\ K_* & K_{**} \end{bmatrix}\right) \tag{5}$$

Since it has been proved that $y_* \mid y$ is developed from a Gaussian multivariate distribution with an average of $K_* K^{-1} y$ and variance of $K_{**} - K_* K^{-1} K_*^T$, the approximate average and variance of the predicted output are obtained as follows:

$$\begin{aligned} E(y_*) &= K_* K^{-1} y \\ \mathrm{var}(y_*) &= K_{**} - K_* K^{-1} K_*^T \end{aligned} \tag{6}$$

After determining meta-parameters of kernel function, parameters of the model including k and σ_n can be determined by Bayesian inference. After training, the GPR model can be used to predict unknown values by the known input values.

3.2. Evaluation parameters of GPR model

In this study, the RMSE (Root Mean Squared Error), MAE (Mean Absolute Error), MAPE (Mean Absolute Percentage Error), MAD (Mean Absolute Deviation), and R^2 (coefficient of determination) parameters have been used to evaluate the performance of GPR modeling. These parameters can be determined as follows:

$$RMSE = \sqrt{\frac{1}{N}\sum_{i=1}^{N}(m_i - p_i)^2} \tag{7}$$

$$MAE = \frac{1}{N}\sum_{i=1}^{N}\left|m_i - p_i\right| \tag{8}$$

$$MAPE = \frac{1}{N}\sum_{i=1}^{N}\left|\frac{m_i - p_i}{m_i}\right| \times 100 \tag{9}$$

$$MAD = median\left(\left|pi - median\ (p)\right|\right) \tag{10}$$

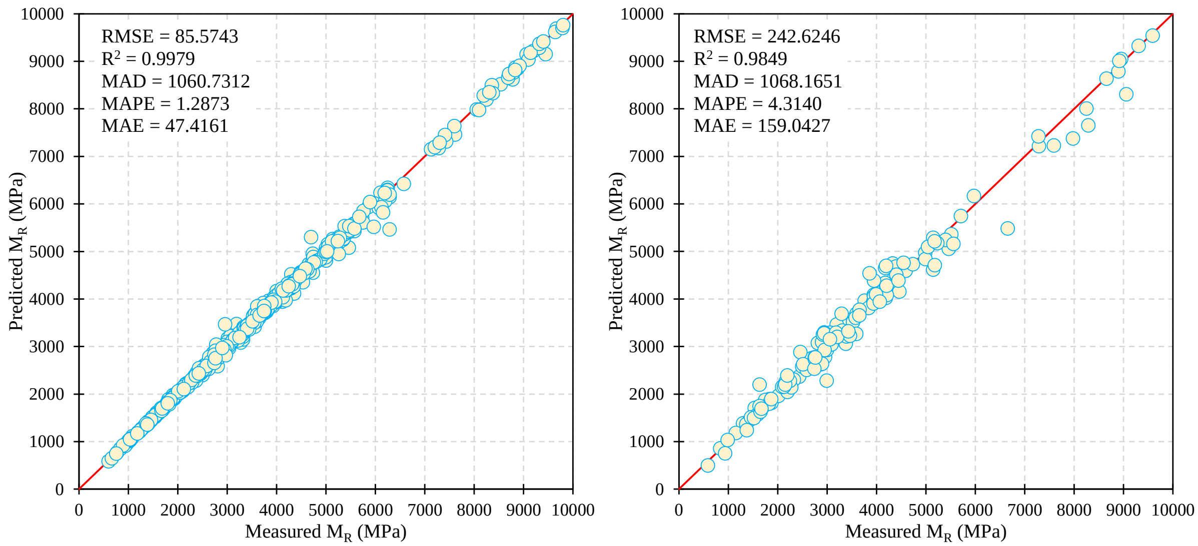
$$R^2 = \left[\frac{N\sum_{i=1}^{N} m_i p_i - \sum_{i=1}^{N} m_i \sum_{i=1}^{N} p_i}{\sqrt{N\sum_{i=1}^{N} m_i^2 - \left(\sum_{i=1}^{N} m_i\right)^2}\sqrt{N\sum_{i=1}^{N} p_i^2 - \left(\sum_{i=1}^{N} p_i\right)^2}}\right]^2 \tag{11}$$

Where N is the number of data to evaluate the desired method, m_i is the measured value for i^{th} data point, and p_i is the predicted value for i^{th} data point.

4. Results and discussion

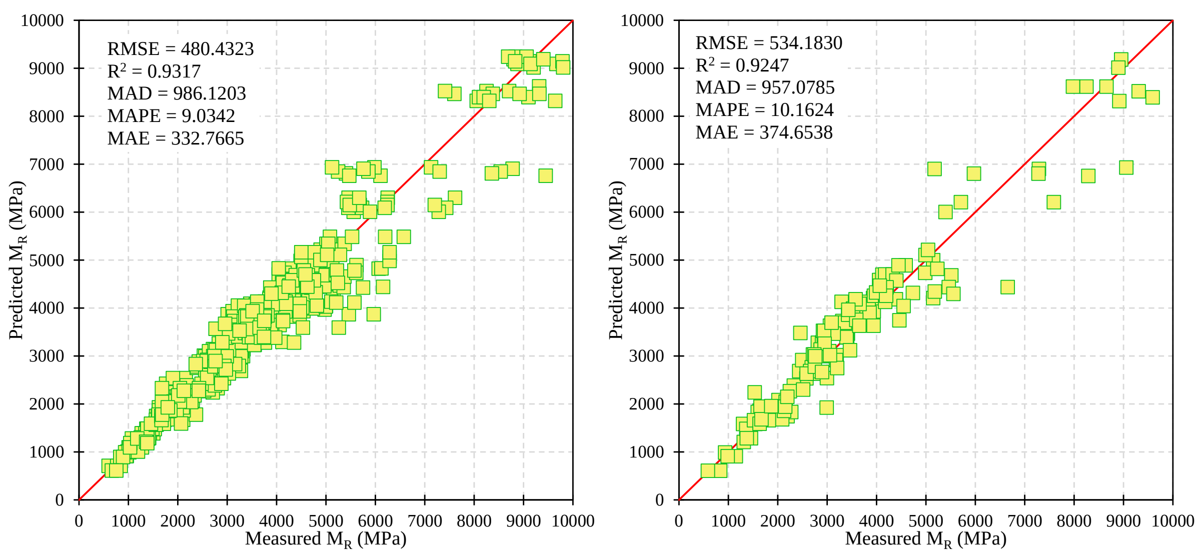
For modeling by the GRP method, 80% and 20% of all dataset records have been considered as training and testing set, respectively. Training and testing sets were randomly selected from the total data points. For this purpose, the data were shuffled in the Excel program and then the first 80% of the data points were selected as the training set and the second 20% of the data points were considered as a testing set. The linear basis function has been used for modeling by GPR method and model overfitting was controlled by considering 10% of the training set as the validation set. Exponential kernel function and linear basis function also were assumed for developing final models. These two functions was selected based on the try and error method and after evaluation of different functions. be In this study four GPR models including model “A” considering all 5 input variables, model “B” considering 4 input variables (by omitting σ_d), model “C” considering 4 input variables (by omitting σ_3), and model “D” considering 3 input variables (by omitting σ_d and σ_3) were developed. Performance of these four GPR models is demonstrated in Figure 2 to 5.

As can be seen, sequence of four models developed in terms of accuracy (R^2 and RMSE) is as A>C>B>D. In fact, model “A” with 5 input variables and values of R^2 and RMSE equal to 0.998 and 5858.336 for the training set and 0.985 and 6427.365 for the testing set has the best performance and model “D” with 3 inputs and values of R^2 and RMSE equal to 0.922 and 7631.497 for the training set and 0.912 and 1373.576 for the testing set has the lowest performance among the four developed models.



Train Test

Fig. 2. The predicted versus measured resilient moduli based on the train and test datasets for model “A”.



Train Test

Fig. 3. The predicted versus measured resilient moduli based on the train and test datasets for model “B”.

Also omitting σ_d variable from input parameters in model “B”, lead to more reduction in model accuracy compared to omitting the σ_3 variable in model “C” which confirms that the resilient modulus of the stabilized base material is more dependent on σ_d parameter compared to the σ_3

parameter. The approximate equality of the coefficient of determination (R^2) for the training and testing set in all cases indicates the high generalizability of the developed models.

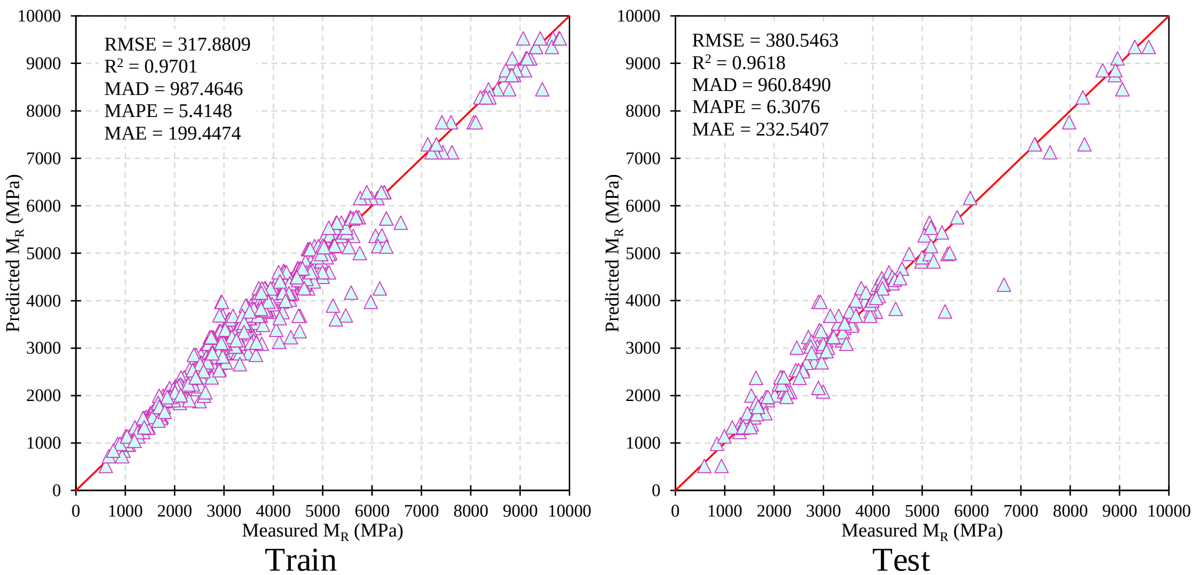


Fig. 4. The predicted versus measured resilient moduli based on the train and test datasets for model “C”.

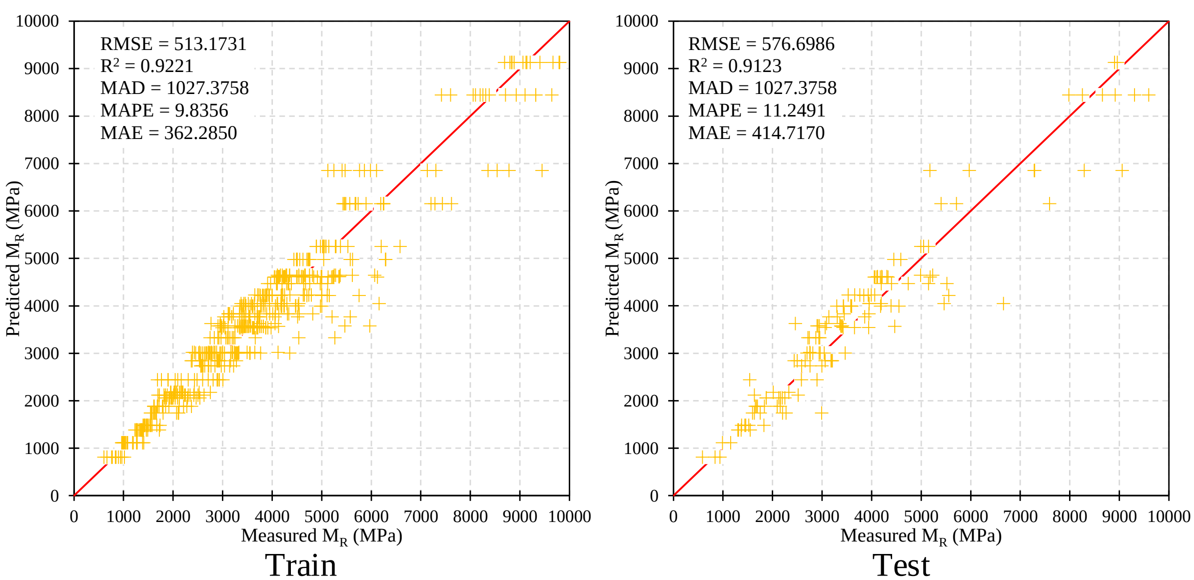


Fig. 5. The predicted versus measured resilient moduli based on the train and test datasets for model “D”.

5. Comparing of performance with other methods

As mentioned in section 2, the dataset used in the present study was adopted from the results of Maloof et al. (2012) [20]. In their research, in addition to conducting laboratory studies, they presented a model for predicting the resilient modulus using the support vector machine (SVM) method. Comparison of the results obtained from the present study and Maloof et al. (2009) study is provided in Table 1. It is evident that the GPR method is more accurate than the support vector machine method; for example, in case of Model "A", the value of R^2 for the whole dataset using the GPR method is 0.995, and for the SVM method is 0.6400. Table 1 also shows the results of modeling accuracy based on the PSO-ELM method based on the Kaloop et al. (2019) study [32]. As can be seen, depending on the number of input parameters, the accuracy of the GPR method can be more or less than the PSO-ELM method. However, if all input parameters are considered, the accuracy of the GPR model is much higher than the PSO-ELM model, so that the RMSE for the GPR model is less than half of RMSE for the PSO-ELM model.

Table 1
Comparison of the GRP method and previously developed models.

Model	Inputs	GPR (This study)		SVM [20]		PSO-ELM [32]	
		R^2	RMSE	R^2	RMSE	R^2	RMSE
A	WDC, CSAFR, DMR, σ_3, σ_d	0.995	116.894	0.640	1134.592	0.981	253.439
B	WDC, CSAFR, DMR, σ_3	0.931	491.145	0.875	659.750	0.948	415.554
C	WDC, CSAFR, DMR, σ_d	0.968	329.743	0.959	371.309	0.973	304.451
D	WDC, CSAFR, DMR	0.920	525.932	0.900	593.540	0.921	521.080

6. Sensitivity analysis

In this study, the Cosine Amplitude Method (CAM) is employed to determine the degree of importance of each input parameter to predict the resilient modulus using the degree of sensitivity index. The degree of sensitivity index can be calculated by the following equation [39,40]:

$$R_i = \frac{\sum_{j=1}^{n} x_{ij}\, y_j}{\sqrt{\sum_{j=1}^{n} x_{ij}^2 \sum_{j=1}^{n} y_j^2}} \tag{12}$$

Where x_{ij} shows the i^{th} independent variable for the j^{th} data point and y_j shows the dependent variable for the j^{th} data point (respect to x_{ij}). To estimate the relationship between input and output variables, the value of R_i must be close to 1, while R_i with a value of zero virtually eliminates the possibility of extracting a correlation. Figure 6 shows the degree of importance of input variables based on the results from the measured and predicted values of the resilient moduli. As it can be seen, the importance of different parameters can be displayed as DMR > CSAFR > σ_d > σ_3>WDC. In other words, the DMR is the most important parameter and the WDC is the least important parameter for predicting the resilient modulus of the stabilized base materials under W-D cycles. Also the difference of R_i between the predicted and measured values for the resilient modulus for WDC, CSAFR, DMR, σ_3, and σ_d parameters is 0.31%, 0.18%, 0.19%, 0.22%, and 0.22%, respectively which shows the high accuracy of the GPR method to predict the resilient modulus.

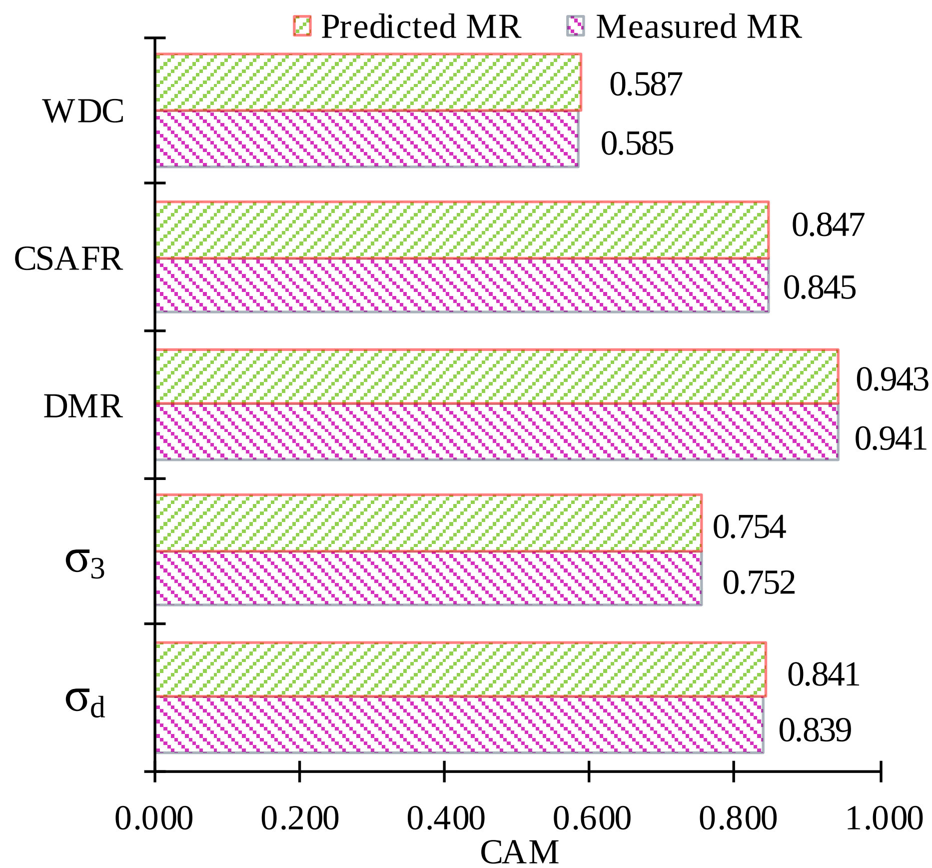


Fig. 6. The importance of each of the variables based on the CAM method.

7. Parametric analysis

Time and cost limitations as well as the limited access to appropriate equipment are generally the main obstacles in laboratory studies. In most cases, investigating the effect of each variable on the test results over a wide range of cases requires fabricating several specimens which are time-consuming and expensive. One of the advantages of modeling is to employ the developed models for parametric studies and evaluating the effect of each input parameter on the model output. As mentioned before, in this study the input parameters are WDC, DMR, CSFAR, confining stress (σ_3), and deviator stress (σ_d).

In this study, by using the optimal GPR model, the effect of interaction between WDC and DMR, the interaction between DMR and CSFAR, the interaction between WDC and CSFAR, and the interaction between σ_3 and σ_d on the resilient modulus of the stabilized base materials have been evaluated. For this purpose, the desired parameter was changed between its minimum value and its maximum value, and other parameters were considered equal to the mean values, and then the resilient modulus was determined according to the change of the desired parameter based on the optimal GPR Model.

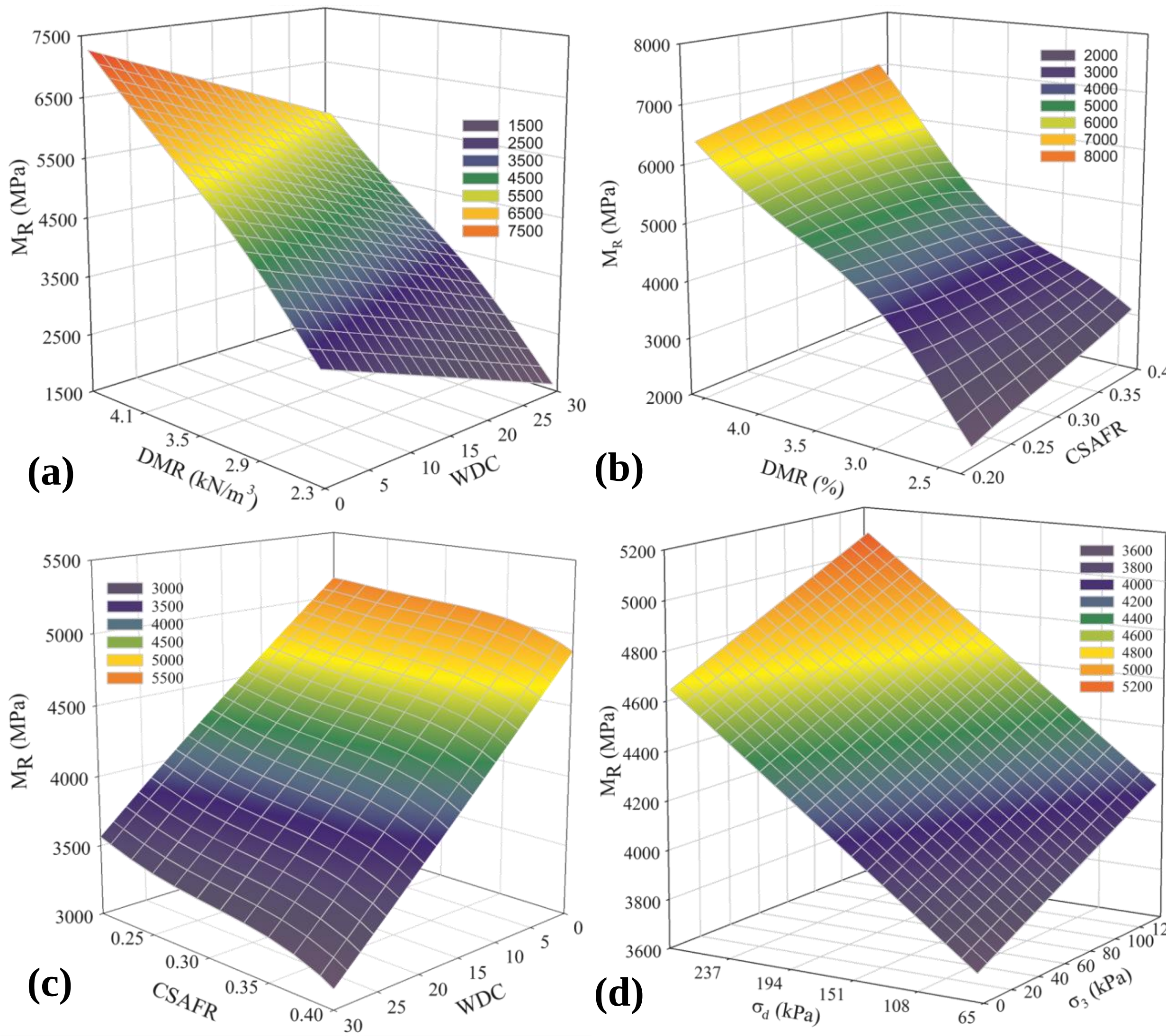


Fig. 7. The parametric analysis of the effect of the input variables on the resilient modulus.

As evident, the resilient modulus dramatically increases as the DMR increases. While the increase in W-D cycles and CSFAR slightly decrease and increase the resilient modulus, respectively. The increase in the confining and deviator stress leads to the increase in the resilient modulus and it can be seen that changes in the deviant stress have more effect on the resilient modulus than the restrictive stress. The interaction of confining and deviator stress shows that increasing these two values increases the resilient modulus and it can be seen that the changes of deviator stress have a greater effect on the resilient modulus than the confining stress.

8. Conclusion

In this study, the GPR modeling method was used to predict the resilient modulus of the stabilized base materials. The results of this research can be summarized as follows:

1- With respect to the values of R^2 and RMSE, model "A" with five inputs and values of R^2 and RMSE respectively equal to 0.998 and 85.574 for the training set and 0.985 and

242.624 for the testing set has the best performance and model "D" with three inputs and values of R^2 and RMSE respectively equal to 0.922 and 513.173 for training set and 0.912 and 576.968 for testing set has the lowest performance.

2- It was found that omitting σ_d variables in model "B" lead to more reduction of model accuracy compared to omitting σ_3 in model "C" which shows a greater dependence of the resilient modulus of the stabilized base materials to the deviator stress compared to the confining stress.

3- The results of sensitivity analysis show that the degree of importance of different input parameters on the resilient modulus is as DMR> CSAFR> σ_d> σ_3> WDC.

4- The difference of degree of sensitivity (R_i) between the predicted and measured values of the resilient modulus for WDC, CSAFR, DMR, σ_3, and σ_d parameters are 0.31%, 0.18%, 0.19%, 0.22%, and 0.22% respectively.

5- Results of the parametric analysis shows that the increase in DMR leads to an increase in the resilient modulus and the increase in the W-D cycles and CSFAR slightly decreases and increases the resilient modulus, respectively. It can also be seen that increasing the deviator stress and the confining stress will increase the resilient modulus of the stabilized base materials.

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