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## An Equation to Determine the Ultimate Flexural Load of RC Beams Strengthened with CFRP Laminates

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### ABSTRACT

In this paper, a new relationship is presented for determining the ultimate flexural load of reinforced concrete beams strengthened with CFRP laminates. An artificial neural network with a suitable performance was used to estimate this equation. First, a collection of laboratory results including 83 data was collected from valid references. This database was then divided into three groups of 51, 16, and 16, which were used to train, validation, and test the proposed equation, respectively. The final model had eleven inputs including concrete compressive strength, width of beam, effective depth, area of tension reinforcement, area of compression reinforcement, yield strength of steel, modulus of elasticity of steel, modulus of elasticity of CFRP sheet, width of CFRP sheet, total thickness of CFRP sheets and, length of CFRP sheet, which were applied to the network to determine the ultimate flexural load as the output of the model. The obtained results from the proposed relationship showed that it was able to use as a predictive equation for the considered target.

## 1. Introduction

Today, FRP materials are widely used for retrofitting. FRP materials can use in three types including layers, grainy material and also bars. The retrofitting effects of FRP plate instead of bars was studied by researchers [1]. Some researchers developed the theoretical approaches to study the flexural capacity of FRP concrete beams [2]. Previous studies have shown that the

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performance of FRP materials is suitable for increasing the capacity of structural elements and for this reason, their use in the building industry is expanding. Finite elements method, which is considered as a suitable tool for analyzing the structures [3] and determining the forces of elements, was investigated by Qu *et al.* [4] to estimate failure modes of FRP strengthened concrete beams. In another study, Xu [5] simulated FRP concrete beams by finite elements method and investigated the bearing capacity of RC beams. The moment capacity of RC beams strengthened with FRP based artificial neural networks was predicted by Zatloukal and Konvalinka [6].

Soft computing methods, which are very efficient models, are used for engineering problems. Their application in various fields of civil engineering has also been evaluated by researchers [7–20]. In this paper, neural networks as a soft computing method are considered to propose a new capable equation to predict the ultimate flexural load in reinforced concrete beams strengthened with CFRP laminates. A collection of experimental results is used to train and validate the equation. Details of the method are described and illustrated in the next sections.

## 2. Definition of the database and selected parameters

One of the most important factors in the construction of a neural network is a valid database, which is used to train the network and determine its unknown parameters. In this paper, the authors used the experimental results of RC beams strengthened with FRP that carried out by researchers [21–37]. This database includes 83 data that has been used in three groups of 51, 16 and 16 for training, evaluating and also testing of the proposed model.

In addition, eleven parameters were used as inputs in the presented neural network including concrete compressive strength, width of beam, effective depth, area of tension reinforcement, area of compression reinforcement, yield strength of steel, modulus of elasticity of steel, modulus of elasticity of CFRP sheet, width of CFRP sheet, total thickness of CFRP sheets and also length of CFRP sheet. Details of these parameters are shown in Table 1 and also Fig.1. Based on the database collected for this article, the amounts of maximum and minimum of each parameter were reported in the table.

**Table 1**

The considered parameters for the proposed model.

Variable	Description	Minimum	Maximum
$x_1$	Concrete compressive strength, MPa	18	55.2
$x_2$	Width of beam, mm	100	500
$x_3$	Effective depth, mm	50.8	419
$x_4$	Area of tension reinforcement, mm <sup>2</sup>	71	2413
$x_5$	Area of compression reinforcement, mm <sup>2</sup>	28	1609
$x_6$	Yield strength of steel, MPa	335	590
$x_7$	Modulus of elasticity of steel, GPa	165	201
$x_8$	Modulus of elasticity of CFRP sheet, GPa	11	240
$x_9$	Width of CFRP sheet, mm	25	480
$x_{10}$	Total thickness of CFRP sheets, mm	0.111	6
$x_{11}$	Length of CFRP sheet, mm	1200	4800
$Y$	Ultimate flexural load, kN	16.1	669.3

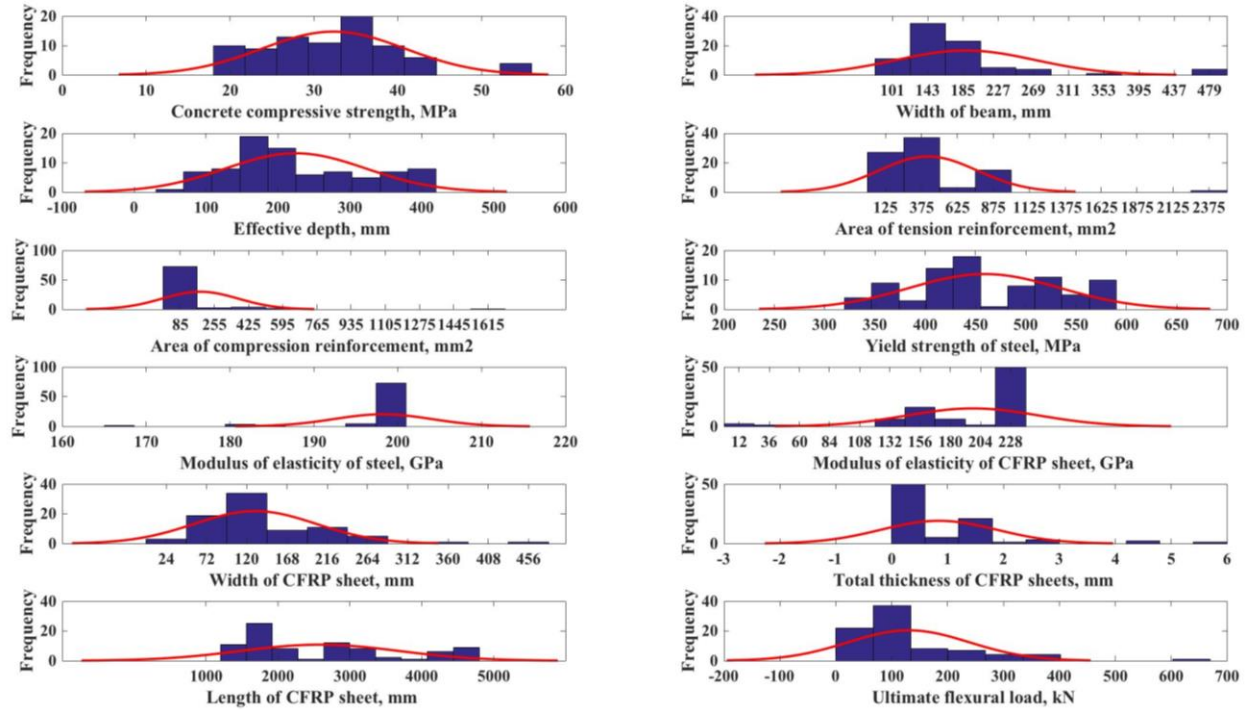


Fig. 1. Details of the selected parameters

In modeling of neural networks, two processes are used as preprocesses of data, which include changing the ordering of data or randomizing, as well as normalizing data or reducing the range of parameter variations. In this paper, the authors used Eq.1 to normalize the database. This relationship limits the range of each of parameters between -1 to +1.

$$X_i = 2 \frac{x_i - x_{mi}}{x_{ma} - x_{mi}} - 1 \quad (1)$$

In this equation, parameters  $X_i, x_i, x_{mi}, x_{ma}$  are normal, experimental, minimum and also maximum value of the variables. Based on this equation, the normal values of input parameters can calculate by Eqs.2.

$$\begin{aligned} X_1 &= 2 \frac{x_1 - 18}{37.2} - 1 & X_2 &= 2 \frac{x_2 - 100}{400} - 1 & X_3 &= 2 \frac{x_3 - 50.8}{368.2} - 1 & X_4 &= 2 \frac{x_4 - 71}{2342} - 1 \\ X_5 &= 2 \frac{x_5 - 28}{1581} - 1 & X_6 &= 2 \frac{x_6 - 335}{255} - 1 & X_7 &= 2 \frac{x_7 - 165}{36} - 1 & X_8 &= 2 \frac{x_8 - 11}{229} - 1 \\ X_9 &= 2 \frac{x_9 - 25}{455} - 1 & X_{10} &= 2 \frac{x_{10} - 0.111}{5.889} - 1 & X_{11} &= 2 \frac{x_{11} - 01200}{3600} - 1 \end{aligned} \quad (2)$$

### 3. Predictive equation

To determine the predictive equation in this paper, the authors used the neural network with a single neuron and one hidden layer. Fig.2 shows the general structure of the mentioned neural network. The parameters  $W$  and  $b$  in this figure are related to weight and bias. The function considered for the node of the hidden layer in this network is a Sigmoid logarithmic function.

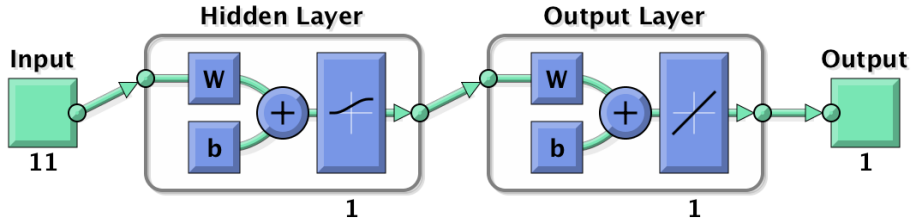


Fig. 2. The structure of the neural network.

Train data is used to train the network and determine the weights and biases of the neural network structure. The results of the training phase are shown in Fig.3 and Fig.4.

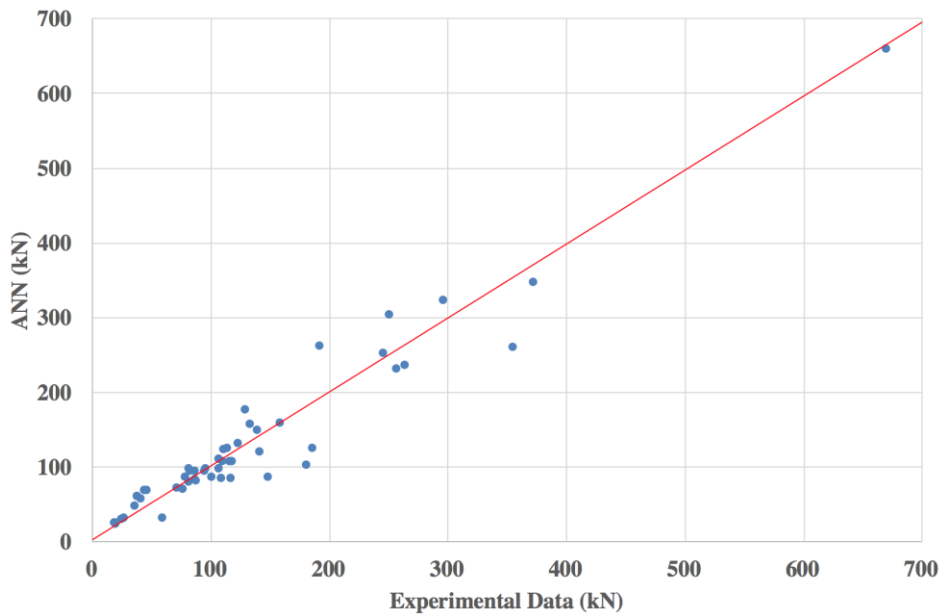


Fig. 3. Scatter plot for training data.

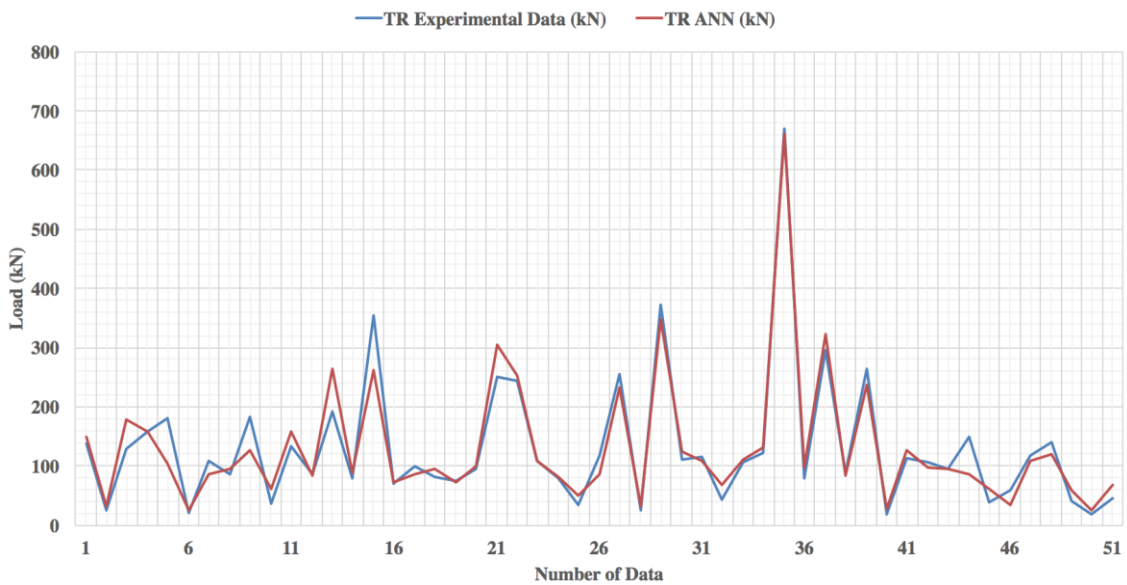


Fig. 4. Results of the proposed network for training data.

After training the network with 51 data intended for training and 16 data for evaluating, the unknown parameters of the network were determined as follows, in which,  $IW$ ,  $LW$ ,  $b_1$  and  $b_2$  were input weights, layer weight, the bias of the hidden layer and also the bias of the output layer.

$$[IW] = \begin{bmatrix} -0.0831 & 0.0974 & -0.4439 & -0.9075 & 0.1203 & -0.1085 & -0.0071 & 0.0412 & -0.3204 & 0.0809 & 0.1325 \end{bmatrix}$$

$$LW = -3.6649$$

$$b_1 = 0.3598$$

$$b_2 = 2.0099$$

To determine the output of the hidden layer, the weight of each input should apply to its corresponding input value. Then, the sum of the resulting values with adding the hidden layer bias (Eq.3) is applied to Sigmoid logarithm function (Eq.4).

$$[X] = \begin{bmatrix} X_1 & X_2 & X_3 & X_4 & X_5 & X_6 & X_7 & X_8 & X_9 & X_{10} & X_{11} \end{bmatrix}$$

$$\begin{aligned} Y_1 &= [IW][X]^T + b_1 \\ &= -0.0831X_1 + 0.0974X_2 - 0.4439X_3 - 0.9075X_4 + 0.1203X_5 - 0.1085X_6 - 0.0071X_7 \\ &\quad + 0.0412X_8 - 0.3204X_9 + 0.0809X_{10} + 0.1325X_{11} + 0.3598 \end{aligned} \quad (3)$$

$$Y_2 = -3.6649 \left( \frac{1}{1 + e^{-Y_1}} \right) + b_2 \quad (4)$$

Based on the above equations and the obtained results of the coefficients, Eq.5 is proposed as the predictive model for determining the ultimate flexural load of reinforced concrete beams strengthened with CFRP laminates.

$$Y (kN) = 326.6(Y_2 + 1) + 16.1 \quad (5)$$

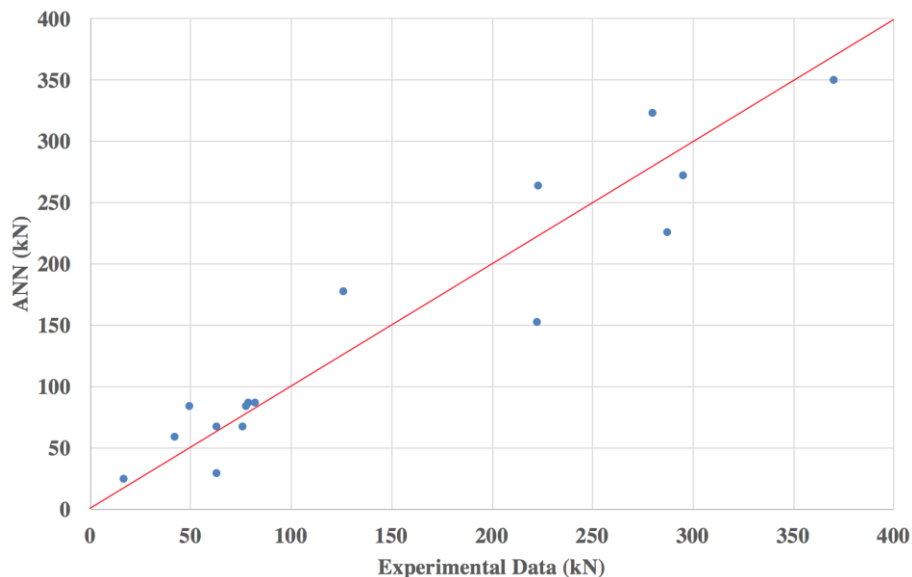


Fig. 5. Scatterplot for checking data.

As mentioned in the previous sections, to evaluate the proposed equation of this research, a collection of 16 datasets. The results of this datasets are presented in Fig. 5 and Fig. 6 which are indicated that the model had reasonable outputs against its corresponding targets.

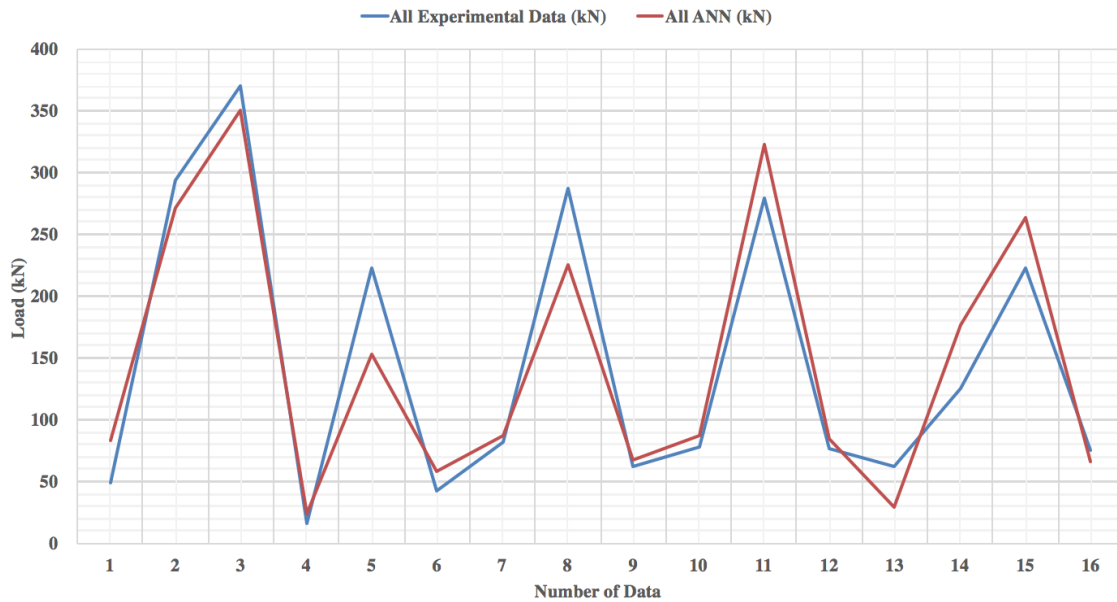


Fig. 6. Results of the proposed network for checking data.

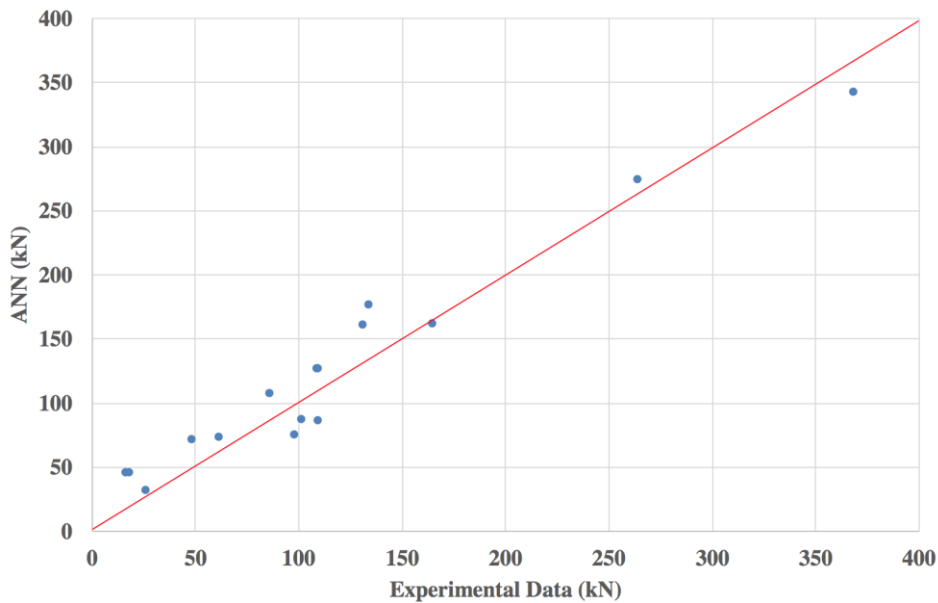


Fig. 7. Scatter plot for testing data.

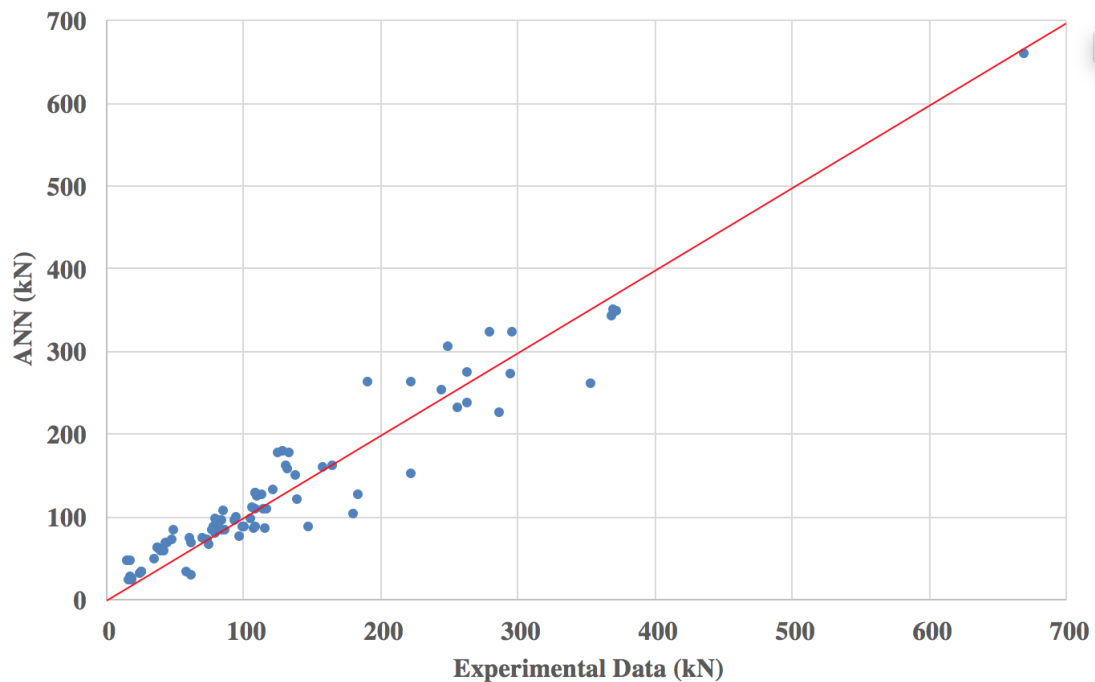
#### 4. Discussion

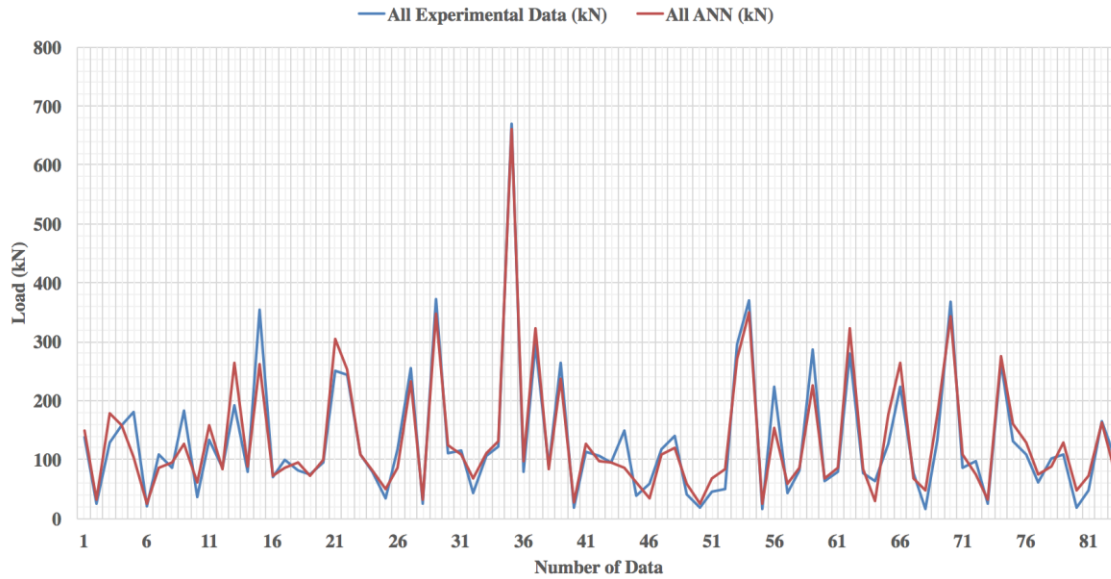
In this section, the ability of the Eq.5 is investigated by 16 datasets as test data. To do this, the outputs of this datasets were calculated and presented in Fig. 7 and Fig. 8. The figures indicated that the proposed equation was evaluated and tested with results of very close to the

experimental results. For all 83 laboratory data, Fig. 7 and Fig. 10 showed the outputs of the presented equation and indicated that the errors are poor and the correlation coefficient between predicted values and laboratory results is also very close. Therefore, the results obtained in this paper show that Eq.5 is suitable for estimating the selected output.



**Fig. 8.** Results of the proposed network for testing data.



**Fig. 9.** Scatter plot for all data.**Fig. 10.** Results of the proposed network for all data.

## 5. Conclusion

In this paper, a relationship is proposed for estimating the ultimate flexural load in reinforced concrete beams strengthened with CFRP laminates. The effective parameters on the target, which are used as inputs of the proposed model, are the concrete compressive strength, width of beam, effective depth, area of tension reinforcement, area of compression reinforcement, yield strength of steel, modulus of elasticity of steel, modulus of elasticity of CFRP sheet, width of CFRP sheet, total thickness of CFRP sheets and, length of CFRP sheet. The unknown coefficients of the proposed equation are driven using a neural network model as well as the results of the laboratory tests. Also, the proposed Equation has good accuracy, which indicates that this relationship can predict the considered output parameter.

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