Optimization of Concrete Beams Reinforced with GFRP Bars

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ARTICLE INFO	ABSTRACT
	Members with GFRP bars exhibit different behavior from those with steel bars due to the brittle and low elastic modulus of the GFRP bars. However, there are limited studies considering the optimum design of reinforced concrete members with GFRP bars compared with extensive studies for reinforced concrete members with steel bars. This study highlighted the performance of reinforced concrete beams with GFRP bars considering the optimum design. The behavioral flexural resistances involving compression, tension, and combined controls are incorporated in the formulation of the design constraints. Also, constraints including deflection serviceability limit states as well as construction requirements are considered. The optimization process is conducted using a genetic algorithm. Comparison with a conventional design is conducted by considering simply supported GFRP reinforced concrete beams in which the efficiency of the developed optimum design has been demonstrated. Analysis results show that tension-controlled sections govern the optimum design despite their brittle performance and highest reduction in strength. However, by increasing design restrictions including depth limits and deflection limits, tension-controlled sections became unable to provide sufficient strength and serviceability and the optimum design was shifted to a more ductile combined resisting control and then to compression-controlled sections.

1. Introduction

The main task of the designer is to design the structure to satisfy both strength and serviceability requirements with a minimum cost. Reducing cross-sectional area and reinforcing area in reinforced concrete members are the main factors that contribute to reducing the cost of the structure. Steel has unrivaled widespread all materials as reinforcement in reinforced concrete structures for about 100 years due to its superior characteristics like the strength and compatibility with concrete. However, the vulnerability of concrete structures reinforced by steel bars to deterioration in aggressive conditions mainly due to corrosion of reinforcing steel bars has led towards seeking non-corroded reinforcing materials. The interest in adopting nonmetallic reinforcing materials in the construction field has been increased in the last three decades. Among these materials, glass fiber reinforced polymer (GFRP) materials have emerged as an alternative reinforcing material that was evolved as a result of extended efforts and research in the field of plastic and composite materials. However, the higher cost of GFRP bars than that of their counterpart steel bars was the main obstacle that prevented designers and clients from using such materials in many construction projects [1]. In contrast, the corrosion of steel bars in harsh conditions and related deterioration of reinforced concrete structures is a major concern in many countries which incur additional annual repair and maintenance costs [2]. Therefore, using GFRP bars as a substitute for steel bars provides the best efficient alternative to avoid corrosion problems and excessive maintenance costs in aggressive environments [3,4]. Usually, the conventional design of reinforced concrete members involves trail and error selection of member dimensions and determining the reinforcement area without any reference to the cost. However, many studies have been conducted and models were developed in the field of cost optimum design of structures. These studies focused on steel reinforced members and many models for optimum design of steel reinforced concrete beams were developed and presented in the literature. Among these studies, Al-Salloum and Siddiqi [5] presented a model for cost optimum design of singly steel reinforced concrete rectangular beam according to the ACI code provisions. In their model, the optimum values of steel ratio and beam depth were obtained by the extermination of a Lagrangian cost function that includes concrete cost, reinforcing steel cost and formwork cost. Also, design curves were presented in terms of optimum steel area and beam depth for various cost ratios. Malasri et al. [6] and Coello and Farrera [7] developed an optimum design model for steel reinforced concrete beams using genetic algorithms. Barros et al. [8] presented an optimum cost design model for singly and doubly steel reinforced concrete beams considering the stress-strain diagram defined in EC2-2001. Zheng et al. [9] developed a model for optimum design of steel reinforced concrete composite beams and used sequential unconstrained minimization technique to solve the problem. Fedghouche and Tiliouine [10] developed a procedure for optimum cost design of reinforced concrete T-beams using the generalized reduced gradient technique. It is obvious that developing design models and studies related to the optimum design of steel reinforced concrete beams are broadly covered. On the other hand, the GFRP reinforced concrete beams exhibit different behavior from that traditional steel reinforced concrete beams due to the unique mechanical properties of GFRP bars that are characterized by high strength, brittle and low elastic modulus. In the last three decades, experimental tests and studies have been carried out to investigate and understand the behavior

of GFRP reinforced concrete beams [11–17]. It was demonstrated that GFRP reinforced concrete beams perform differently from their counterpart steel reinforced concrete beams in the two recognized flexural modes of failure including compression and tension controls. It was noticed that the tension control failure (failure corresponding to GFRP bars rupture) is more brittle and catastrophic than the compression control failure (failure corresponding to concrete crushing). In addition, it was found that GFRP reinforced concrete beams are vulnerable to large deflection due to the low elastic modulus of the GFRP bars. Therefore, design codes and standards prefer compression control modes over brittle tension control modes. Also, rigorous limits on beam depth to confine excessive deflection of GFRP reinforced concrete members were specified [18,19]. Burgoyne and Balfas [1] presented design curves for GFRP reinforced and pre-stressed concrete beams in terms of optimum GFRP area and beam depth for various cost ratios following the procedure developed by Al-Salloum and Siddiqi [5]. However, Balfas and Burgoyne [20] considered flexural strength constraint corresponding to the balanced and compression modes of failure and neglected brittle tension failure.

In this study, the cost optimum design of GFRP reinforced concrete beams is developed and investigated. Three zones of ultimate strength including compression control (crushing of concrete), transition zone and tension control (rapture of bars) are specified for designing reinforced concrete beams [18]. Also, strength reduction factors are specified for each performance of the beam within the three zones. This study investigates the zone of the ultimate strength that leads to the optimum design considering the effect of different design constraints. Constraints covering flexural strength and deflection limit are formulated by design provisions specified by the ACI 440.1R-15 [18]. Also, constraints including construction requirements are considered in the presented scheme.

2. GFRP vs. steel reinforced concrete beam

The flexural strength of reinforced concrete beams involves either compression control demonstrated by concrete crushing (specified 0.003 strain at extreme compression fiber) or tension control demonstrated by yielding of steel reinforcing bars or rupture of FRP bars in steel reinforced concrete beams and GFRP reinforced concrete beams, respectively. ACI 318-19 [21] favors ductile tension control in steel reinforced concrete beams over the compression control brittle failure. It is obvious that in steel reinforced concrete beams, ACI 318-19 [21] specifies lower strength reduction for tension controlled beams compared to higher strength reduction for compression controlled beams. In contrast, GFRP reinforced concrete beams exhibit more brittle behavior corresponding to tension control (rupture of GFRP bars) than those corresponding to compression control (crushing of concrete). Accordingly, ACI 440.1R-15 [18] specifies higher strength reduction for tension controlled beams compared to lower strength reduction for compression controlled beams. The flexural reduction factors of reinforced concrete members according to ACI 318-19 [21] and ACI 440.1R-15 [18] are illustrated in Table 1. In addition, GFRP reinforced concrete beams exhibit large deflections compared to steel reinforced concrete beams due to the relatively low modulus of elasticity of GFRP bars compared to that of the steel bars. To overcome these shortcomings, ACI 440.1R-15 [18] specifies more rigorous depth limits for FRP reinforced concrete beams than those specified by ACI 318-19 [21]. Table 2 compares

total depth limits to control deflection as specified by both ACI 318-19 [21] and ACI 440.1R-15 [18].

Table 1
The flexural reduction factors for reinforced concrete members.

Mode of failure	Steel reinforced concrete beams [21]	FRP reinforced concrete beams [18]
Compression control	0.65	0.65
Transition zone	0.65-0.9	0.55-0.65
Tension control	0.9	0.55

Table 2
Minimum total depths of reinforced concrete beams.

Support condition	Minimum thickness	
	Steel reinforced concrete beams [21]	FRP reinforced concrete beams [18]
Simply supported	l/16	l/10
One end continuous	l/18.5	l/12
Both ends continuous	l/21	l/16
Cantilever	l/8	l/4

3. Optimum design of FRP beams

3.1. Objective function

In this study, examples of simply supported beams are considered. Fig. 1 illustrates design variables of a typical rectangular singly reinforced concrete simply supported beam. Therefore, the objective function to minimize the cost of singly reinforced concrete rectangular beams with GFRP bars is considered.
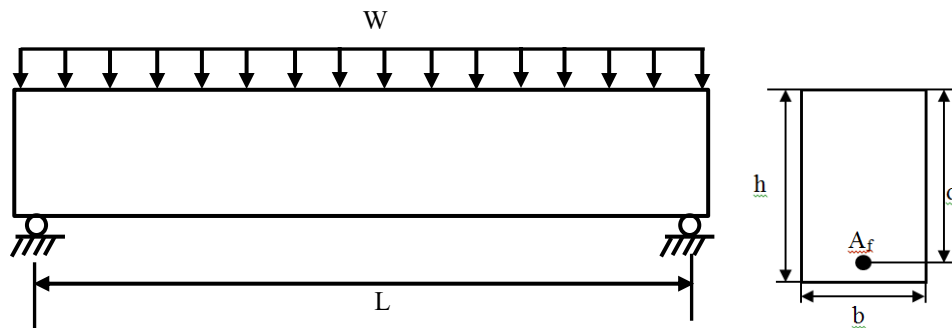
Fig. 1. Layout and cross section of a typical rectangular singly reinforced concrete simply supported beam.

The total cost of the beam includes the cost of concrete, GFRP bars, formworks and labor cost. In this study, the labor and material transportation cost are implicitly considered within material cost. The mathematical form of the objective function is given by

$$C = C_f A_f \gamma_f + C_c bh + C_r(b + 2h) + C_s A_v \gamma_s L/s \tag{1}$$

where C = total cost of the beam section; C_f = unit weight cost of GFRP bars; C_c = unit volume cost of concrete; C_r = unit area cost of the formwork; A_f = the area of GFRP reinforcing bars; b = width of the beam; h = total depth of the beam; and γ_f = mass density of the GFRP reinforcing bars.

3.2. Constraints

Design constraints are formulated considering both ultimate and serviceability requirements for the design of GFRP reinforced concrete beam according to ACI 318-19 [21] and ACI 440.1R-15 [18] codes provisions.

Flexural strength constraint

The design constraint incorporating ultimate flexural strength is defined by

$$M_u - \emptyset M_n \leq 0 \tag{2}$$

where M_u = ultimate factored moment that obtained from structural analysis considering imposed factored loads; M_n = nominal flexural strength of GFRP reinforced concrete beam; and $\emptyset$ = flexural strength reduction factor.

The flexural capacity depends on the mode of failure involving crushing of concrete, rupture of GFRP reinforcing bars or a combination of both. The failure mode is distinguished according to the GFRP reinforcement ratio (ρ_f) compared to the balanced reinforcement ratio (ρ_{fb}) that expressed as

$$\rho_f = \frac{A_f}{bd} \tag{3}$$

and

$$\rho_{fb} = 0.85 \, \beta_1 \frac{f_c'}{f_{fu}} \frac{E_f \varepsilon_{cu}}{E_f \varepsilon_{cu} + f_{fu}} \tag{4}$$

where f_c' = ultimate concrete strength (MPa); f_{fu} = ultimate tensile strength of the GFRP bars (MPa); ε_{cu} = ultimate concrete strain at extreme fibers that is usually set equal to 0.003; E_f = modulus of elasticity of GFRP bars (MPa); and the factor β_1 is given by

$$\beta_1 = 0.85 - 0.05 \frac{f_c' - 28}{7} \geq 0.65 \tag{5}$$

The compression control sections are corresponding to the reinforcement ratio greater than the balanced ratio ($\rho_f > \rho_{fb}$) in which failure mode is governed by concrete crushing and the nominal flexural strength of the section is defined by

$$M_n = \rho_f f_f \left(1 - 0.59 \frac{\rho_f f_f}{f_c'}\right) bd^2 \tag{6}$$

where

$$f_f = \left(\sqrt{\frac{(E_f \varepsilon_{cu})^2}{4} + \frac{0.85 \beta_1 f_c'}{\rho_f} E_f \varepsilon_{cu}} - 0.5 E_f \varepsilon_{cu}\right) \leq f_{fu} \tag{7}$$

On the other hand, tension control is associated with the reinforcement ratio less than the balanced ratio ($\rho_f < \rho_{fb}$) in which the failure mode is governed by the rupture of GFRP bars and the nominal flexural strength of the section is given by

$$M_n = \rho_f f_u \left(1 - \frac{\beta_1}{2} \frac{\varepsilon_{cu}}{\varepsilon_{cu} + \varepsilon_{fu}}\right) bd^2 \tag{8}$$

where ε_{fu} = ultimate strain of the GFRP bars.

Flexural strength reduction factors specified by ACI 440.1R-15 [18] are illustrated in Table 1. ACI 440.1R-15 [18] specifies the limits of the compression control like $\rho_f \geq 1.4 \rho_{fb}$ while tension control limits like $\rho_f \leq \rho_{fb}$. In addition ACI 440.1R-15 [18] specifies a strength reduction factor that varies linearly between 0.55 and 0.65 for the transition zone corresponding to the reinforcement ratio between ρ_{fb} and $1.4 \rho_{fb}$.

Shear strength

According to ACI 318-19 [21], shear strength is resisted by beam cross section at;

$$V_u - \emptyset V_c \leq 0 \tag{9}$$

The nominal shear strength of the section without shear reinforcement is defined by

$$V_c = 0.083 \sqrt{f_c'} \, bd \tag{10}$$

When the shear forces exceed the shear strength and lie in the range given by;

$$0.083\emptyset \sqrt{f_c'} \, bd \leq V_u \leq 0.17\emptyset \sqrt{f_c'} \, bd$$

Minimum shear reinforcement is required.

$$V_u - \emptyset(V_c + A_v f_y d/s) \leq 0 \tag{11}$$

Where s is the spacing between stirrups. Using steel bars with 10 mm diameter for stirrup provides area of reinforcement A_v equal to 142 mm^2 to provide minimum reinforcement yields a reinforcement at distance (s) equal to lesser of d/2 and 600 mm. Then, shear constraint becomes.

$$V_u - \emptyset(V_c + \max(2A_v f_y, A_v f_y d/600)) \leq 0 \tag{12}$$

Otherwise, when ultimate shear stress exceeded cross section shear strength, stirrups are arranged at distance equal to lesser of d/4 and 300 mm and shear constraint is.

$$V_u - \emptyset(V_c + \max(4A_v f_y, A_v f_y d/300\,)) \leq 0 \tag{13}$$

Minimum reinforcement constraint

ACI 440.1R-15 [18] recommends minimum FRP reinforcement to avoid premature rupture of FRP bars in which it is expressed as

$$A_{f,min} = \frac{0.41\sqrt{f_c'}}{f_{fu}} b_w d \geq \frac{2.3}{f_{fu}} b_w d \tag{14}$$

Therefore, the minimum reinforcement constraint becomes

$$A_f - A_{f,min} \geq 0 \tag{15}$$

Deflection limit constraint

ACI 318-19 [21] specifies immediate deflection (Δ_{iL}) and long terms deflection (Δ_{LT}) using the following equations

$$\Delta_{iL} = (\Delta_i)_{LL+DL} - (\Delta_i)_{DL} \tag{16}$$

and

$$\Delta_{LT} = (\Delta_i)_{uns\,LL} + \lambda[\,(\Delta_i)_{DL} + 0.2(\Delta_i)_{LL}\,] \tag{17}$$

$(\Delta_i)_{LL}$= deflection due to live load alone; $(\Delta_i)_{DL}$= deflection due to dead load alone; $(\Delta_i)_{LL+DL}$= deflection due to live and dead loads; $(\Delta_i)_{uns\,LL}$= deflection due to unsustained load and λ= parameter associated to long term deflection that equal to 0.6 ξ for singly reinforced concrete beams and ξ is taken equal to 1 and 2 for duration short duration and for more than five years, respectively [21]. Also, ACI 318-19 [21] specifies short term and long term deflection limits. Accordingly, short term and long term deflection constraints are expressed by

$$\Delta_{iL} - \Delta_{st\,limit} \leq 0 \tag{18}$$

and

$$\Delta_{LT} - \Delta_{lt\,limit} \leq 0 \tag{19}$$

where $\Delta_{st\,limit}$ and $\Delta_{lt\,limit}$ = short term and long term deflection limits, respectively.

Depth to width ratio constraints

In the design of reinforced concrete beams, it is common in practice to specify the upper and lower limits of beams total depth to width ratio. Typically values of beams total depth to width ratio (h/b) vary between 1 and 3. The beams total depth to width ratio constraints are considered in the developed model using the following equations for the lower and upper limits, respectively

$$b - h < 0 \tag{20}$$

$$h - 3b < 0 \tag{21}$$

Maximum total depth

Arising from construction requirements, maximum total depth of the beam h_{max} is commonly defined. In this study maximum total depth of the beam is considered using the following inequality

$$h - h_{max} < 0 \tag{22}$$

4. Optimization method (Genetic algorithm)

Genetic algorithms have been successfully implemented in broad applications of structural optimization [22–25]. Genetic algorithm is an iterative computer based method that is routinely adopted for solving stochastic search and optimization problems [26]. The genetic algorithm follows the survival of the fittest individuals in generating the successive populations that mimic the biological evolution. From the search space, genetic algorithm starts by selecting the individuals in a random form of the initial population (solution). The individuals represent the parents of the individual children for the following generation. Based on the objective function corresponding to the optimization problem, the genetic algorithm evaluated the potential solutions in the current population and the fitness value is assigned. Creating children population process involves parent selection, crossover, mutation and acceptance. Parents are selected based on their fitness and then parents mutated and crossover is processed to create children individuals. Then parents are interchanged randomly and the process of creating children is repeated until achieving the best fitness value of individuals (children) that will be selected to constitute the following generation. The process of generating successive populations (generations) by the genetic algorithm continues until satisfying the stop criteria. Genetic algorithm uses various stopping criteria including specified number of generations, processing time limit or the tolerance limits corresponding to improved solution. Accordingly, genetic algorithm steps include;

Initialization

A group of chromosomes is randomly selected within the boundary of the problem. Genetic algorithms deal with representative codes of the problem parameters (solutions) rather than directly deal with parameters themselves in which individual solutions or chromosomes are coded using vectors of components (genes) linked to problem parameters. Thus, the first step of genetic algorithm is the construction of a set of possible values of design parameters randomly generated within the ranges of the problem constraints. All possible values of parameters are encoded to form genes that linked together to form the individuals (solutions) in terms of strings of values referred to as chromosomes. Usually, chromosomes are coded using binary forms in which each form represents different solution. For instance, two selected chromosomes with nine binary variables are randomly represented such as; Chromosome1 100110110 and Chromosome2 100011110.

Selection

The second step is the selection of parent chromosomes from the initial generation. The selected parents are those of higher potential fitness (here the higher fitness represents the lower costs) than others depending on fitness selection method. The process of choosing potential best fit set of parent chromosomes from the initial generation using a specific method refers to a selection process. The selection process is repeated in each successively developed generation until the termination at the last generation according to the assigned termination criteria. More details about selection method can be found in [26].

Crossover

The crossover is the operation that randomly cut genes from chromosomes of parent generation and exchange them with other genes from other chromosomes from the same parent generation to produce new chromosomes that represent children generation. Therefore, the developed children chromosomes consist of shared genes from two parent chromosomes.

Mutation

Mutation is the process that follows crossover that randomly changes one or several genes within a chromosome to produce new children in order to avoid premature convergence that may leads to local optima. The process is conducted by replacing selected bits with 0 by 1 and vice versa that corresponding to binary encoding of chromosomes.

Termination

Steps from second to forth are repeated until the process is terminated. Subsequently, the population develops from generation to generation, deficient individuals are gradually disappeared and efficient individuals are survived and optimized. Processing genetic algorithm by MATLAB [27] adopts four termination criteria including number of limited generations (the default is100 times the number of defined variables), limits of processing time, attaining a defined value of best fitness. Otherwise, the process terminates at the generation after several successive unchanged values of fitness (stall generations).

Satisfying constraint

In the constraint optimization including genetic algorithm technique, two types of solutions are developed including feasible solutions and infeasible solutions. Feasible solutions satisfy constraints conditions while infeasible solutions violate constraints conditions. However, the optimum solution occurs at the boundary between the two solutions. Penalty function is adopted to handle the solution at the boundary between the infeasible and feasible solutions. The infeasible solutions are penalized based on the degree of violation. However, both feasible and close infeasible solutions are used for reproduction of children generation. In this study, the modeling and the analysis are conducted using MATLAB R software [27]. The flow chart of genetic algorithm operations is depicted in Fig. 2.
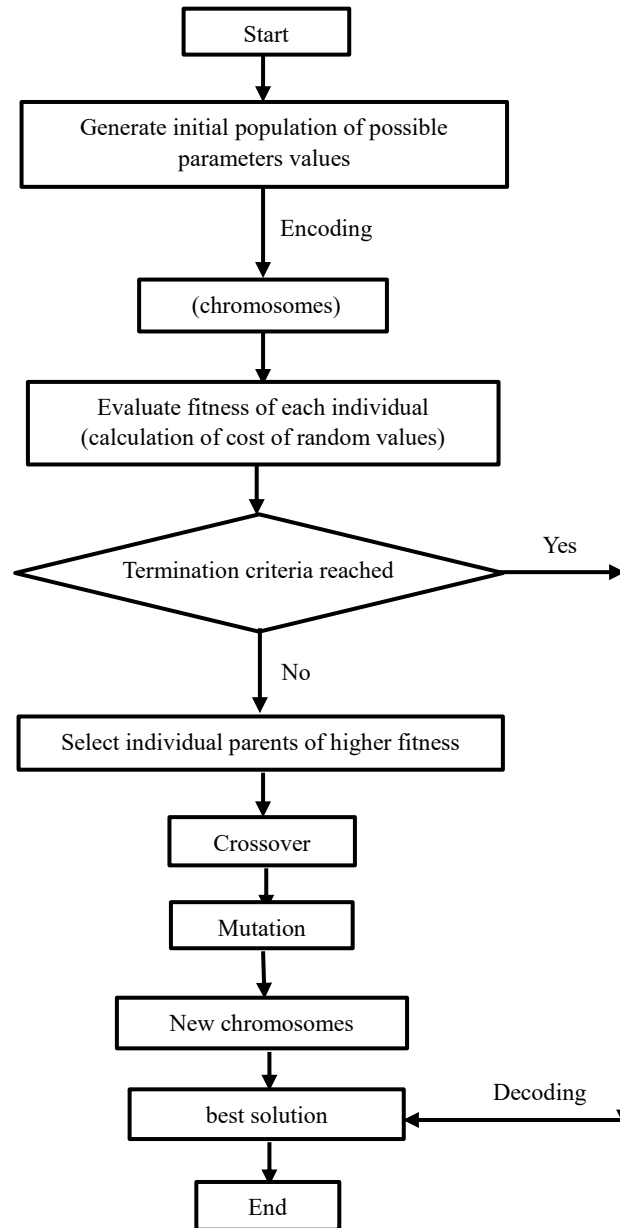
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                        ┌──────────┐
                        │  Start   │
                        └────┬─────┘
                             ↓
              ┌──────────────────────────────┐
              │ Generate initial population of possible │
              │       parameters values        │
              └───────────────┬──────────────┘
                              │ Encoding
                              ↓
                    ┌──────────────────┐
                    │  (chromosomes)   │
                    └────────┬─────────┘
                             ↓
              ┌──────────────────────────────┐
              │ Evaluate fitness of each individual │
              │ (calculation of cost of random values) │
              └───────────────┬──────────────┘
                              ↓
                    ◇ Termination criteria reached ◇ ── Yes ──┐
                              │ No                            │
                              ↓                               │
              ┌──────────────────────────────┐               │
              │ Select individual parents of higher fitness │  │
              └───────────────┬──────────────┘               │
                              ↓                               │
                       ┌────────────┐                         │
                       │ Crossover  │                         │
                       └─────┬──────┘                         │
                             ↓                                │
                       ┌────────────┐                         │
                       │  Mutation  │                         │
                       └─────┬──────┘                         │
                             ↓                                │
                    ┌──────────────────┐                      │
                    │ New chromosomes  │                      │
                    └────────┬─────────┘                      │
                             ↓                  Decoding       │
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                    │  best solution   │
                    └────────┬─────────┘
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                       ┌──────────┐
                       │   End    │
                       └──────────┘
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Fig. 2. Genetic algorithm operations flow chart.

5. Results

Two examples are presented to evaluate the validity and efficiency of the presented cost optimum design procedure using the genetic algorithm. The first example involves a comparison between the optimal design obtained from the presented procedure and the conventional design of a typical model presented and designed by ACI 440R-06 [28]. The effect of varying costs of materials on the optimal design compared to conventional design is considered. The second example involves a parametric investigation to study the sensitivity of the optimal design to the effect of strength and construction constraints demonstrated by the beam depth as well as the

effect service deflection limits on the optimum design of the concrete beams reinforced with GFRP bars.

Example 1

A typical GFRP reinforced concrete rectangular beam conventionally designed by ACI 440.1R-06 [28] committee is selected in this study to compare the design results obtained from the proposed optimum design with that of the conventional design. The main change on the ACI 440.1R-06 [28] is the deflection calculations. In this study, deflection calculations are repeated to satisfy ACI 440.1R-15 [18]. However, similar design results are obtained in which the design was governed by the flexural limits. The beam is a simply supported interior beam spanning (L) over 3.35 m and carrying superimposed dead load and service live load of 3 kN/m and 5.8 kN/m (20% sustained), respectively. The design requirements entail that the beam deflection does not exceed $l/240$. Also, the depth of the beam is limited to 356 mm due to construction restrictions The compressive strength of the concrete $f_c' = 27.6$ MPa. The GFRP bars have tensile strength $f_{fu} = 620.6$ MPa, rupture strain $\varepsilon_{fu} = 0.014$ and modulus of elasticity $E_f = 44800$ MPa. Design variables include beam width B= $178\ mm$, beam thickness H $\leq 356\ mm$ and reinforcement ratio
ρ_f varied between the minimum value of 0.0037 and maximum assumed value of 0.01. In this study, a continuous variation of design parameters is assumed.

In this study, the optimization model has been run several times to investigate best values of genetic algorithm parameters that provide stable and optimum solution. The number of generations is varied between 100 and 500, number of stall generations varied between 20 and 100, number of population number varied between 4 and 42 and tolerance values have been varied between default value 10^{-6} and 10^{-15} in which the stable solution has been investigated. A stable solution has been achieved at number of population equal to 26, number of generations greater than 10, stall generations equal to 20 and tolerance value of 10^{-12}. Figs. 3 and 4 illustrate the investigation results considering the effect of number of populations and generations on the optimum design; respectively.
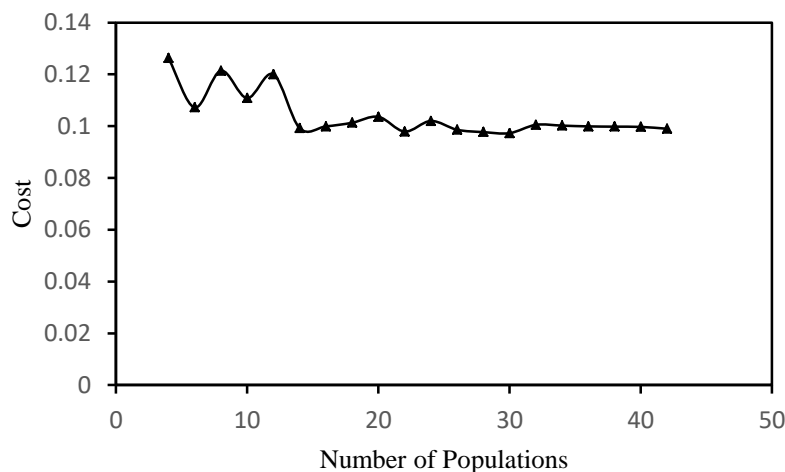
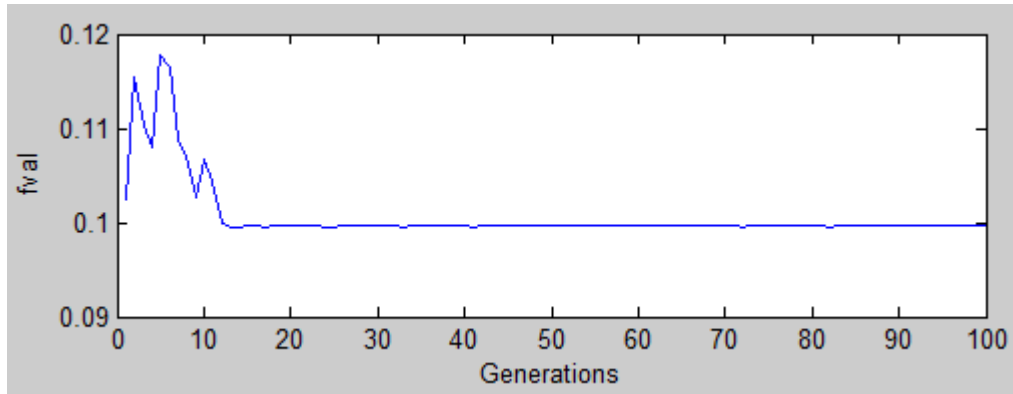**Fig. 3.** Variation of cost with population number.

Fig. 4. Variation of cost with number of generations.

On the other hand, analysis results are compared with conventional design. Table 3 shows comparison results between the conventional design and the optimum design considering cost ratios of GFRP bars to concrete (C_f/C_c) equal to 150 and formwork to concrete (C_{fw}/C_c) equal to 0.5. Also, values of design variables and cost considering three resisting modes, tension control, compression control and combined control are illustrated in Table 3. The values of design variables include dimensions and reinforcement ratio as well as the total estimated cost.

Table 3
Comparison of the design obtained from the developed model and conventional procedure.

Design	b (mm)	h (mm)	ρ_f	Cost	Cost reduction %
ACI 440-R [28]	178	305	0.0128	0.1399Cc	----
Optimal design (Concrete crushing control)	178	326	0.008	0.1283Cc	8.29
Optimal design (Transition zone)	178	326	0.0057	0.1084Cc	22.5
Optimal design (GFRP bars rupture control)	178	356	0.0038	0.0997Cc	28.7

It is obvious that considering different failure controls resulted in different design details and costs despite of that each failure control meets ACI 440.1R-15 [18] provisions. It is common that the cost of materials is varied around the world and even within the country. Cost ratios include GFRP bars cost to concrete cost ratio (C_f/C_c) and formwork cost to concrete cost ratio (C_{fw}/C_c) are adopted in this study to illustrate the sensitivity of the optimum design with varying materials costs. To investigate the effect of varying cost ratios (C_f/C_c) on design results and the total cost, the cost ratio (C_f/C_c) is varied including the values of 100, 150, 200 and 250. However, formwork cost to concrete cost ratio (C_{fw}/C_c) is maintained at constant value of 0.5. The optimum design results including values of design variables and cost considering the effect of cost ratio (C_f/C_c) are illustrated in Table 4. Also, the effect of varying cost ratio on the conventional and optimum design considering different resistance modes is shown in Fig. 5. Design variables presented in Table 4 represent optimum design of the beam corresponding to the behavior within transition zone. Table 4 shows that the increases in cost ratio leads to increase in total cost despite that the design variables related to optimal design are not altered. Also, the required reinforcement ratio corresponding to the optimum design is close to the lower bond (value of 0.0038) of the transition zone. The other two resisting controls behave in a similar trend.

Table 4
Optimal cost design results considering effect of varying cost ratio Cf/Cc.

	Cf/Cc	100	150	200	250
Optimum design	bopt (mm)	178	178	178	178
	hopt (mm)	353	356	356	356
	ρ_{fopt}	0.0038	0.0038	0.0038	0.0038
	Copt	0.0875 Cc	0.0997 Cc	0.1117 Cc	0.1236 Cc
Conventional design	bconv (mm)	178	178	178	178
	dconv (mm)	305	305	305	305
	ρ_{fconv}	0.0128	0.0128	0.0128	0.0128
	Cconv	0.1116 Cc	0.1399 Cc	0.1686 Cc	0.1970 Cc
Cost reduction (%)		21.6	28.7	33.7	37.2
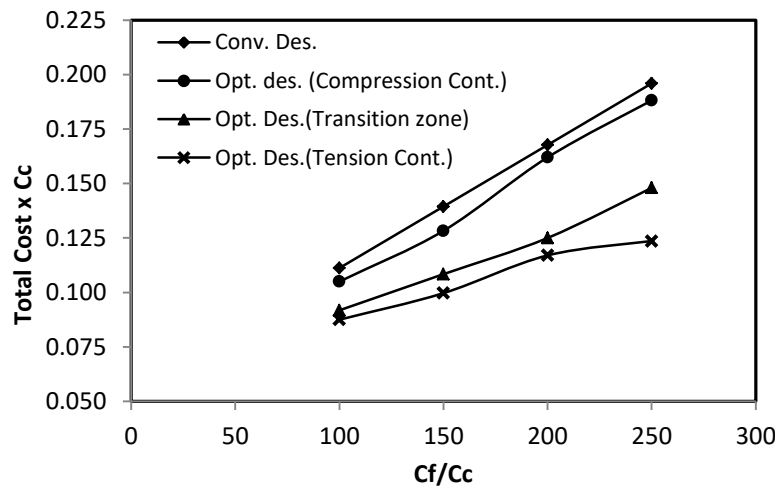
Fig. 5. Effect of cost ratio (Cf/Cc) on the total cost corresponding to different behavior.

Example 2

In this example, a simply supported interior beam spanning over 5 m and carrying superimposed dead load and service live load of 15 kN/m and 9 kN/m (20% sustained), respectively is considered. The compressive strength of the concrete $f_c' = 30$ MPa. The GFRP bars have tensile strength $f_{fu}= 496$ MPa, rupture strain $\varepsilon_{fu}= 0.014$ and modulus of elasticity $E_f= 41000$ MPa. The influence of the limit of the beam total depth on the optimum design is studied considering the beam total depth equal to 450 mm, 500 mm, 550 mm, 600 mm and 650 mm. The short term and long term deflection limits of $l/180$ and $l/240$, respectively are assumed. The GFRP bars cost to concrete cost ratio (C_f/C_c) of constant value equal to 150 is considered. Design variables include beam width ranged from 300 mm to 500 mm , reinforcement ratio ρ_f varied between the minimum value of 0.0046 and maximum assumed value of 0.04 and beam total depth varied from 300 mm to maximum limits considering values equal to 450 mm, 500 mm, 550 mm, 600 mm and 650 mm as upper limit of the parametric investigation. A continuous variation of design parameters is assumed. The variation of cost with the number of developed generations is illustrated in Fig. 6 for the case of depth limit equal to 550 mm. It is obvious that after thirty generations, a stable optimum solution has been achieved. On the other hand, Table 5 and Fig. 7 illustrate the effect of total depth limits on the optimum design values.
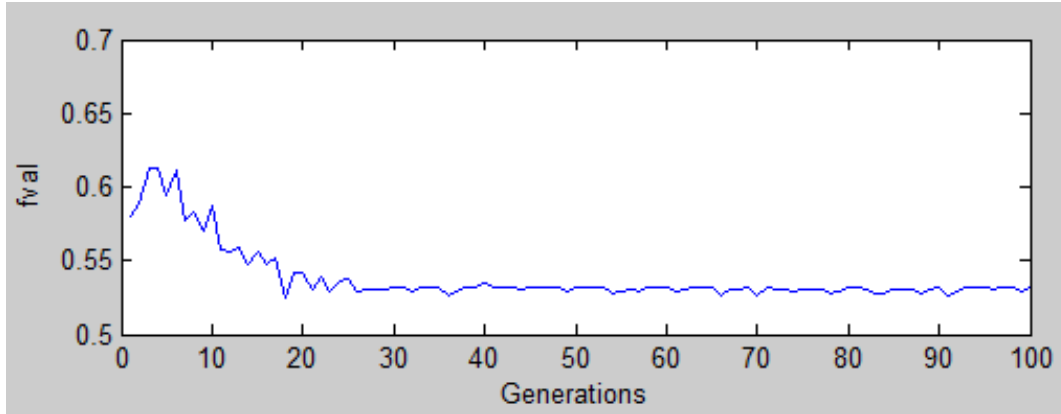
Fig. 6. Variation of cost with number of generations for depth limit equal to 550 mm.

Table 5
Optimum cost design results considering effect of varying total depth limits.

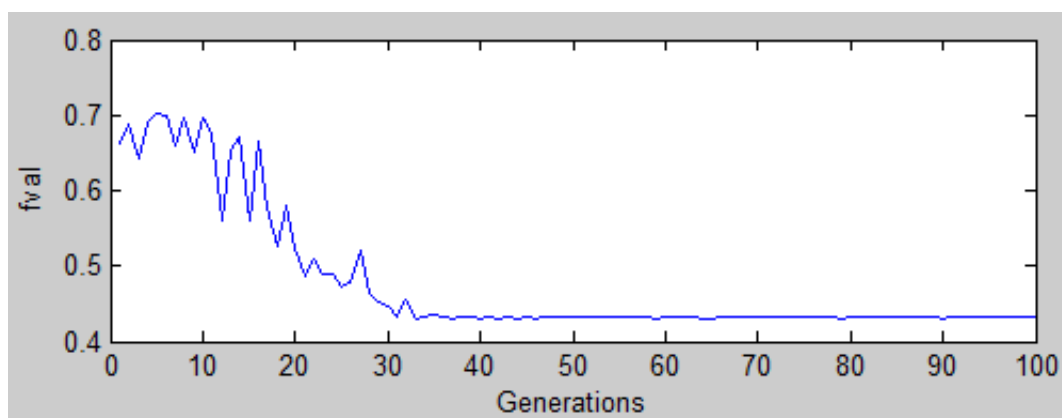| Failure control		Total depth limit (mm)				
		450	500	550	600	650
Compression control	bopt (mm)		427	363	361	300
	hopt (mm)	-------	498	550	560	625
	ρ_{fopt}		0.0154	0.0124	0.0113	0.0113
	Copt		0.652 Cc	0.534 Cc	0.5144 Cc	0.481 Cc
Transition zone	bopt (mm)			404	333	340
	hopt (mm)	-------	------	545	600	610
	ρ_{fopt}			0.0107	0.0093	0.0081
	Copt			0.540 Cc	0.453 Cc	0.439 Cc
Tension control	bopt (mm)				386	318
	dopt (mm)	-------	------	------	600	650
	ρ_{fopt}				0.007	0.0071
	Copt				0.453 Cc	0.410 Cc
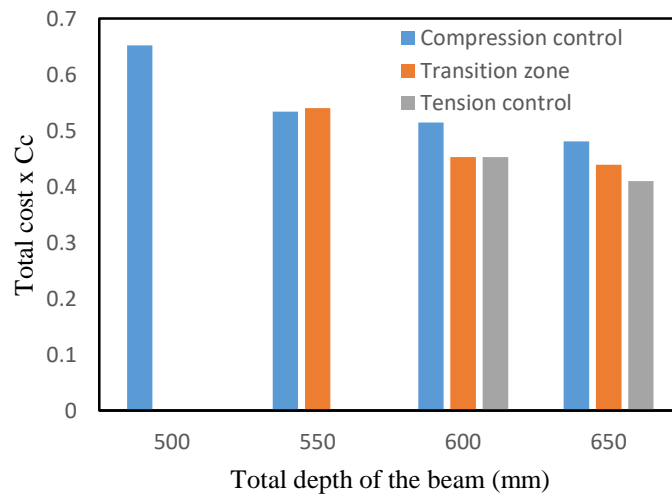
Fig. 7. Effect of total depth limits on the total cost corresponding to different beam behavior.

It is shown that increasing restriction on the beam depth has a significant influence on the optimum design and the total cost of the beam. It is obvious that decreasing total depth limit from 650 mm to 600 mm results in increase in the total cost of about 10.5 %, 3.2 %, and 6.9 % corresponding to tension control, transition zone and compression control, respectively. Also, it shows that the optimum cost corresponding to tension control is significantly influenced by varying the limits of the beam total depth compared to the compression control. However, the optimum cost corresponding to the combined control has been slightly affected. It is clear that tension-control resistance governs the minimum cost in beams with a total depth limit of 650 mm while both the tension-controlled section and the section at the transition zone have similar costs corresponding to total depth limit of 600 mm. Further decreasing the thickness limit to 550 mm resulted in the inability of tension control to provide the required strength for the beam section and the required strength is only achieved by compression control resistance. Also, decreasing total depth limit to 500 mm results in inability of both combined control (transition zone) and tension control to provide the required strength for the beam section and the required strength is only achieved by compression control resistance with increase in cost of about 35.6 % over that section with a total depth of 650 mm. Eventually, the section failed in resisting the applied loads by any resisting mode when the total depth limit decreased to 450 mm as illustrated in Table 5.

On the other hand, the effect of assigning constant beam width on the optimum design is studied. The design variables include constant beam widths equal to 350 mm, 400 mm, 450 mm and 500 mm are considered, a reinforcement ratio ρ_f varied between the minimum value of 0.0046 and maximum assumed value of 0.04 and assumed beam total depth varied from 300 mm to 900 mm. Also, a continuous variation of design parameters is assumed. The variation of cost with the number of developed generations is illustrated in Fig. 8 for the case of defined beam width equal to 350 mm. It is obvious that after thirty-five generations, a stable optimum solution has been achieved. On the other hand, Table 6 and Fig. 9 illustrate the influence of beam width on the optimum design values.


Fig. 8. Variation of cost with number of generations for assigned beam width equal to 350 mm.

Table 6
Optimum cost design results considering effect of varying limits of beams width.

Failure control		Width limit (mm)			
		350	**400**	**450**	**500**
Compression control	**bopt (mm)**	350	400	450	500
	hopt (mm)	792	843	814	824
	$\boldsymbol{\rho_{fopt}}$	0.012	0.012	0.012	0.012
	Copt	0.79 Cc	0.967 Cc	1.08 Cc	1.18 Cc
Transition zone	**bopt (mm)**	350	400	450	500
	hopt (mm)	758	744	734	691
	$\boldsymbol{\rho_{fopt}}$	0.0086	0.0086	0.0086	0.0087
	Copt	0.627 Cc	0.702 Cc	0.800 Cc	0.826 Cc
Tension control	**bopt (mm)**	350	400	450	500
	dopt (mm)	730	699	690	641
	$\boldsymbol{\rho_{fopt}}$	0.0047	0.0047	0.0047	0.0047
	Copt	0.454 Cc	0.501 Cc	0.555 Cc	0.573 Cc
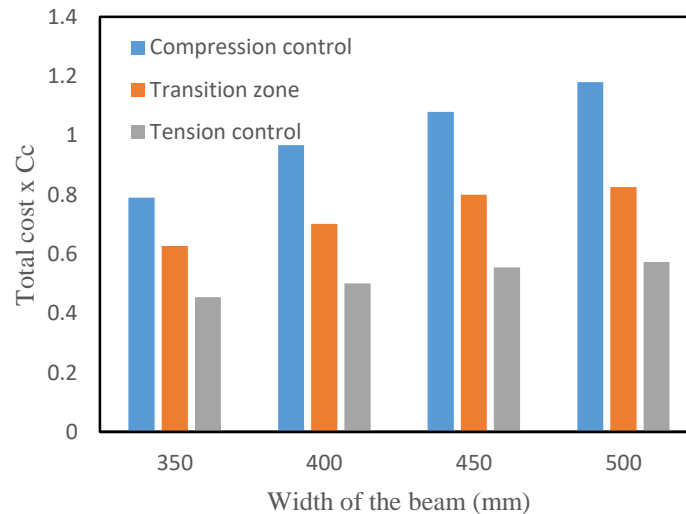
Fig. 9. Effect of defined width on the total cost corresponding to different beam behavior.

It is shown that using specified larger widths leads to larger sections with increased total cost of the beam. It is obvious that increasing width of the beam from 350 mm to 500 mm results in increase in the total cost of about 26 %, 32 %, and 49 % corresponding to tension control, combined control and compression control, respectively. It is obvious that the influence of increasing beam width on the optimum cost corresponding to tension control is less than that corresponding to the combined control and compression control, respectively. Also, it is clear that tension-control resistance governs the minimum cost in beams corresponding to all defined beam widths.

Further investigations considering the influence of the deflection limits are conducted. The cost optimum design is computed for various long term deflection limits including values of $l/240$, $l/360$, $l/480$ and $l/560$. A constant total depth limit of 650 mm is adopted. Design variables include beam width ranged from 300 mm to 500 mm , reinforcement ratio ρ_f varied between the minimum value of 0.0046 and maximum assumed value of 0.04 and assumed beam thickness

varied from 300 mm to 900 mm. Also, a continuous variation of design parameters is assumed. Also, the GFRP bars cost to concrete cost ratio (C_f/C_c) of constant value equal to 150 is considered. The variation of cost with the number of developed generations is illustrated in Fig. 10 for the case of deflection limit equal to l/360 mm. It is obvious that after forty generations, a stable optimum solution has been achieved.

The optimum design values using the presented procedure considering various deflection limits and corresponding to different resistance modes are illustrated in Table 7 and Fig. 11. Results show that the optimum design values and the corresponding minimum cost have varied with varying deflection limits. Also, it is obvious that the optimum design corresponding to tension control is significantly influenced by varying deflection limits compared to the combined control and compression control. The total cost per meter length of the beam increases about 18.3 %, 14.8 %, and 6.9 % corresponding to tension control, combined control and compression control, respectively by varying the deflection limits from $l/240$ to $l/360$. It is noted that the increase in cost results from the increase in section dimensions with higher values in tension control than those in combined control and in compression control. However, adopting more rigorous deflection limits equal to $l/480$ and $l/560$ result in the failure of the tension control section to satisfy the design requirements.
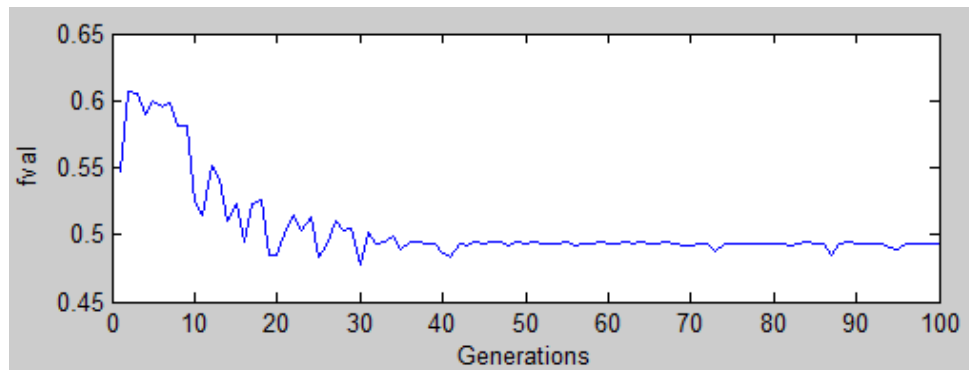

Fig. 10. Variation of cost with number of generations for the case of deflection limit of L/360.

Table 7
Optimum cost design results considering effect of varying deflection limits.

Failure control		Deflection limit			
		l/240	l/360	l/480	l/560
Compression control	bopt (mm)	300	308	328	357
	hopt (mm)	625	650	650	640
	ρ_{fopt}	0.0113	0.0113	0.0142	0.0144
	Copt	0.481 Cc	0.514 Cc	0.630 Cc	0.679 Cc
Transition zone	bopt (mm)	340	335	384	390
	dopt (mm)	610	650	650	650
	ρ_{fopt}	0.0081	0.0096	0.0098	0.0104
	Copt	0.439 Cc	0.504 Cc	0.586 Cc	0.617 Cc
Tension control	bopt (mm)	318	387	-----	-----
	dopt (mm)	650	650	-----	-----
	ρ_{fopt}	0.0071	0.0068	-----	-----
	Copt	0.410 Cc	0.485 Cc	-----	-----
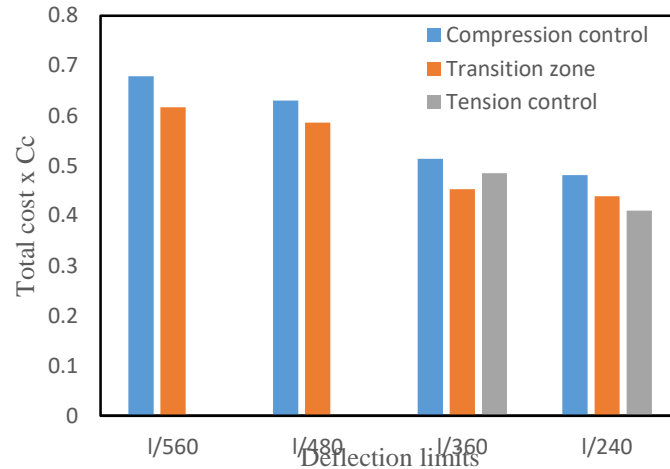
Fig. 11. Effect of deflection limits on the total cost corresponding to different beam behavior.

The effect of shear constraint on the performance of the design model is investigated by conducting the analysis with and without shear constraint. Further investigations were conducted by increasing loads. Analysis results of the considered models show that the shear constraint have negligible influence on the optimum design and equations 9 and 10 have been satisfied. In contrast, the effect of shear constraint becomes tangible with increasing load. In this study, only results of models that govern by flexural constraints are presented.

5. Conclusion

In this study, cost optimization of concrete beams singly reinforced by GFRP bars based on ACI 440.1R-15 [18] and ACI 318-19 [21] codes is presented and investigated. Concrete beams reinforced by GFRP bars exhibit different ultimate strength performance including crushing of concrete, rapture of bars and balanced failure (transition zone) depending on the values of the design variables. This study estimates the values of design variables corresponding to different failure performance that lead to optimum cost design. Also, the effect of different design constrains on the optimum design has been investigated in this study. The design procedure involves determining section dimensions, required reinforcement area of GFRP bars corresponding to the optimum cost while satisfying the provisions and limitations of the ACI 440.1R-15 [18] and ACI 318-19 [21] codes. The objective function is formulated by including the cost of the GFRP bars, concrete and formwork. A Series of constrains is formulated including strength behavior, serviceability limits, design requirements specified by ACI 440.1R-15 [18] and ACI 318-19 [21] codes as well as construction restrictions. Beam flexural strength controlled by either concrete crushing, GFRP bars rupture or combined failure is included in the formulations of the strength constraints. The optimization is processed using genetic algorithms. Typical examples are presented in order to demonstrate the validity and efficiency of the presented approach in which the results are compared with the conventional design. Also, parametric investigations are conducted to identify the effect of different binding constraints on the optimum cost design. The following conclusions are observed from the results of the analysis:

1-   Optimum cost design is clearly achieved by using the presented approach compared to the conventional design.

2-   Adopting the presented cost optimization approach increases the level of integrity of the design in which the design requirements are processed systematically and automatically.

3-   The high GFRP cost to concrete cost ratio result in tendency of the required reinforcement area to the lower bond of each resistance control in cases with relaxed restrictions and the least cost is corresponding to the tension control that governs the optimum design despite the specified highest strength reduction corresponding to this resisting mode.

4-   By increasing design restriction, the tension control mechanism becomes unable to provide sufficient strength in which resistance is shifted to the transition zone and compression control.

5-   The sensitivity analyses have demonstrated that the optimum design is remarkably influenced by deflection limits as well as the strength limits.

6-   The optimum design corresponding to tension control is more sensitive to varying design parameters than that corresponding to the combined control and that corresponding to the compression control.

Finally, this study can be extended by adopting discrete variation of parameters in the modeling and studying optimum design of continuous beams, slabs and frame structures with FRP reinforcement bars.

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Conflicts of interest

The authors declare no conflict of interest.

Authors contribution statement

Thaer, MSA: Conceptualization; Data curation; Formal analysis; Investigation; Methodology; Resources; Software; Validation; Visualization; Roles/Writing – original draft; Writing – review & editing.

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