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Slope Stability Evaluation Using Tangent Similarity Measure of Fuzzy Cube Sets

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ABSTRACT

Due to various geological problems and geological materials of the slope, there is a kind of non-continuous and uncertain natural geological body. Because of the complexity of various external factors, slope stability is not easy to be determined, which leads to the ambiguity of human's judgments between stability and instability. Therefore, it is crucial that a simple evaluation method for judging the slope stability with uncertain information is established in slope stability analysis. This study selects nine impact factors: the lithology type, the slope structure, the development degree of discontinuity, the relationship between inclination and slope of discontinuities, the slope height, the slope angle, the mean annual precipitation, the weathering degree of rock, and the degree of human action, which can be expressed as the fuzzy cubic information (the hybrid information of both a fuzzy value and an interval-valued fuzzy number). Then, a tangent similarity measure between fuzzy cube sets (FCSs) is developed for the slope stability evaluation, where the tangent similarity measure values between FCSs of the slope sample and FCSs of slope stability grades/patterns (stability, slight stability, slight instability, and instability) are used for the assessment of the slope stability in FCS environment. Lastly, eight slope samples are provided as the actual cases to show that the eight evaluation results of slope stability using the proposed similarity measure of FCSs are in accordance with the actual results of the eight actual cases, which indicate the effectiveness of the proposed method under FCS environment.

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1. Introduction

The slope is a kind of non-continuous and uncertain natural geological body. Hence, there are uncertain characteristics in the natural slope state due to various geological problems and geological materials of the slope. Hence, uncertainty in slope problems is unavoidable and influenced by tectonics, topography, hydrometeorology, and other factors. The slope stability evaluation methods can be divided into two categories: deterministic method and uncertainty method. Among them, the deterministic method includes the limit analysis method [1,2] and the limit equilibrium method [3–6]. In the 1970s, slope researchers gradually realized the deficiencies of previous deterministic methods in practical engineering. Hence, attempts have been made to evaluate and analyze slope stability using various uncertain methods, and then a large number of uncertain analysis methods have emerged to evaluate slope stability. First, the reliability analysis methods for slope stability analysis [7,8] were proposed based on the knowledge of probability theory and mathematical statistics, where random variables often include material properties of the slope rock mass, slope geometry, and external loads and affect the slope stability. Second, the fuzzy comprehensive evaluation methods for slope stability analysis [9,10] are characterized by fuzzy values, which can better express fuzzy evaluation problems. Since slope stability assessment needs to consider uncertain and incomplete information in an uncertain environment, the reliability analysis and fuzzy comprehensive evaluation methods are difficult to determine the probability distribution or membership function. Subsequently, Liu [11] proposed the theory of uncertainty to describe the inaccuracy of data. Then Zhou [12] uses the uncertain theory to evaluate slope stability under uncertain environment. However, these theories or methods cannot handle such a problem with both partial determinate and partial uncertain information. Furthermore, Zhou et al. [13] introduced a clustering analysis of using a distance-based similarity measure and provided a better classification method for the slope stability analysis.

The classic fuzzy set [14] is expressed by its membership degree in the unit interval [0, 1]. But in the real world, there exists both partial determinate and partial uncertain information. Then, it is difficult to express both certain and uncertain information simultaneously by the classic fuzzy set. However, a fuzzy cubic set (FCS) presented by Jun et al. [15] has attracted much attention since FCS implies its partial certain and partial uncertain information. Hence, FCS can better express the hybrid form of both a fuzzy value and an interval-valued fuzzy number (IVFN). Obviously, FCS is better suited for the representation of its partial indeterminate and partial determinate information in indeterminate and determinate environments. Furthermore, Fu et al. [16,17] further proposed similarity measures of cubic hesitant fuzzy sets and applied them to the initial evaluation of benign prostatic hyperplasia symptoms and the risk grade assessment of prostate cancer in the cubic hesitant fuzzy setting. Unfortunately, FCS has not been studied in its measure algorithm and application in engineering fields so far. However, there may exist partial indeterminate and partial determinate information in actual slope problems, and then in this case existing evaluation methods of slope stability are difficult to express and assess them. It is obvious that we need to develop a new assessment method of slope stability for solving evaluation problems of the actual slope stability with FCS information. To reach this purpose, this original study will establish a tangent similarity measure between FCSs and apply it to slope stability evaluation in both indeterminate and determinate (FCS) environment. Then, by the eight actual samples of the slope obtained from Zhejiang province, China as the actual cases, the evaluation method of slope stability based on the tangent similarity measure of FCSs is used for the slope stability evaluation of the actual samples with FCS information.

The rest of this study is organized by the following. Section 2 presents some basic concepts of FCS and a tangent similarity measure of FCSs. Section 3 gives the evaluation method using the tangent similarity measure of FCSs and applies it to the slope stability evaluation of actual cases. Lastly, conclusions and future work are given in Section 4.

2. Tangent similarity measure of FCSs

FCS was firstly proposed by Jun et al. [15]. This section will introduce the basic concepts of FCS and give the tangent similarity measure of FCSs to be used for the slope stability evaluation in the following study.

An FCS consists of both a fuzzy value and an IVFN. The FCS Z in a universe of discourse X is defined by the following form [15].

$$Z = \{x, A(x), \lambda(x) | x \in X\}$$

where $\lambda(x)$ is a fuzzy value and $A = [A^-(x), A^+(x)]$ is an IVFN for $x \in X$. Then, we define:

(i) $Z = \{x, A(x), \lambda(x) | x \in X\}$ as an internal FCS if $A^-(x) \le \lambda(x) \le A^+(x)$ for $x \in X$;

(ii) $Z = \{x, A(x), \lambda(x) | x \in X\}$ as an external FCS if $\lambda(x) \notin [A^-(x), A^+(x)]$ for $x \in X$.

For the convenient expression, a basic element in Z is denoted as $z = (A_z, \lambda_z) = ([A_z^-, A_z^+], \lambda_z)$, which is named as a fuzzy cubic number (FCN).

Based on the tangent similarity measure of single-valued neutrosophic sets proposed by Ye [18], we can extend it to FCSs to present the tangent similarity measure between FCSs below.

Definition 1. Let two FCSs be $R = \{r_1, r_2, \ldots, r_n\}$ and $H = \{h_1, h_2, \ldots, h_n\}$ in the universe of discourse $X = \{x_1, x_2, \ldots, x_n\}$, where $r_i = (A_{ri}, \lambda_{ri})$ and $h_i = (A_{hi}, \lambda_{hi})$ are two FCNs for $i = 1, 2, \ldots, n$. Thus, the tangent similarity measure of the FCSs R and H is presented as follows:

$$M(R,H) = 1 - \frac{1}{n}\sum_{i=1}^{n}\left\{\tan\left[\frac{\pi}{16}\left(|A_{ri}^- - A_{hi}^-| + |A_{ri}^+ - A_{hi}^+|\right) + \frac{\pi}{8}\left(|\lambda_{ri} - \lambda_{hi}|\right)\right]\right\}. \quad (1)$$

Corresponding to the properties of the tangent similarity measure of single-valued neutrosophic sets [18], the tangent similarity measure of $M(R, H)$ also contains the following properties:

(i) $0 \le M(R, H) \le 1$;

(ii) $M(R, H) = 1$ if and only if $R = H$;

(iii) $M(R, H) = M(H, R)$;

(iv) If N is a FCS in X and $R \subseteq H \subseteq N$, then $M(R, N) \le M(R, H)$ and $M(R, N) \le M(H, N)$.

If we consider $\omega_i \in [0,1]$ as the weight of the elements r_i and h_i with $\sum_{i=1}^{n} \omega_i = 1$, the corresponding weighted tangent measure is given as follows:

$$M_\omega(R,H) = 1 - \sum_{i=1}^{n} \omega_i \left\{ \tan\left[\frac{\pi}{16}\left(\left|A_{ri}^{-} - A_{hi}^{-}\right| + \left|A_{ri}^{+} - A_{hi}^{+}\right| \right) + \frac{\pi}{8}\left(\left|\lambda_{ri} - \lambda_{hi}\right| \right) \right] \right\}. \quad (2)$$

Clearly, the weighted tangent measure of $M_\omega(R, H)$ also contains the following properties:

(i) $0 \le M_\omega(R, H) \le 1$;

(ii) $M_\omega(R, H) = 1$ if and only if $R = H$;

(iii) $M_\omega(R, H) = M_\omega(H, R)$;

If N is an FCS in X and $R \subseteq H \subseteq N$, then $M_\omega(R, N) \le M_\omega(R, H)$ and $M_\omega(R, N) \le M_\omega(H, N)$.

3. Evaluation method of slope stability using the tangent similarity measure of FCSs

In this section, we can use the tangent similarity measure of FCSs for the slope stability evaluation with FCS information.

Based on the geological disaster situations of slope in Zhejiang Province, P. R. China, we consider the slope samples in Zhejiang Province as their actual evaluation cases. According to the main factors of affecting the slope stability in these cases and the actual experience [13,19–24], the set of nine representative impact factors [13,19–24] is presented as $R = \{r_1, r_2, \cdots, r_9\}$, which indicates the set of the lithology type (r_1), the slope structure (r_2), the development degree of discontinuity (r_3), the relationship between inclination and slope of discontinuities (r_4), the slope height (r_5), the slope angle (r_6), the mean annual precipitation (r_7), the weathering degree of rock (r_8), and the degree of human action (r_9). Then, the evaluation patterns of the slope stability are based on the four evaluation grades introduced in [13]: stability (S_1), slight stability (S_2), slight instability (S_3), and instability (S_4). Then, the weight vector $\omega = (\omega_1, \omega_2, \cdots, \omega_9)$ indicates the importance of the nine impact factors. Thus, the specific impact factors of the slope stability for the four evaluation grades are shown in Table 1, where the scoring values of r_1-r_4, r_8, and r_9 are obtained by the scoring from 0 to 10. So, the normalized data (FCSs) in Table 2 are given by the interval value and interval average value (interval expected value) of each impact factor r_k (k = 1, 2, 3, 4, 8, 9) divided by 10, r_5 divided by 150, r_6 divided by 90, and r_7 divided by 200 regarding the data in Table 1.

Regarding the evaluation cases of the slope stability, we select the eight sets of actual slope samples in Zhejiang province, as shown in Table 3. Then by the same normalized calculation/algorithm, the normalized data (FCSs) for the eight groups of actual slope samples Q_s (s = 1, 2, …, 8) are given in Table 4, which are expressed as the eight FCSs of actual slope samples under an FCS environment.

Table 1
Impact factors of the slope stability for the four stability evaluation grades.

Impact factors (r_i)	Stability (S_1)	Slight stability (S_2)	Slight instability (S_3)	Instability (S_4)
Lithology type (r_1)	Hard rock (0-3)	Cemented semi-hard rock (3-6)	Semi-hard rock with poor cementation (6-8)	Soft rock and loose rock soil (8-10)
Slope structure (r_2)	Homogeneous structure (0-3)	Block structure (3-6)	Bedding structure (6-8)	Broken loose and loose structure (8-10)
Development degree of discontinuity (r_3)	Not developing (0-3)	Less developed (3-6)	More developed (6-8)	Very developed (8-10)
Relationship between inclination and slope of discontinuities (r_4)	Reverse slope (0-3)	Flat slope (3-6)	Oblique slope (6-8)	Downhill (8-10)
Slope height (r_5)	0-20m	20-50m	50-100m	100-150m
Slope angle (r_6)	0-10°	10-30°	30-60°	60-90°
Mean annual precipitation (r_7)	0-50mm	50-100mm	100-150mm	150-200mm
Weathering degree of rock (r_8)	Unweathered (0-3)	Weak weathering (3-6)	Moderate weathering (6-8)	Strong weathering (8-10)
Degree of human action (r_9)	Slight (0-3)	Lighter (3-6)	Heavier (6-8)	Weight (8-10)

Table 2
Normalized data (FCSs) of impact factors for the four stability evaluation grades.

r_i	S_1	S_2	S_3	S_4
r_1	([0,0.3],0.15)	([0.3,0.6],0.45)	([0.6,0.8],0.7)	([0.8,1.0],0.9)
r_2	([0,0.3],0.15)	([0.3,0.6],0.45)	([0.6,0.8],0.7)	([0.8,1.0],0.9)
r_3	([0,0.3],0.15)	([0.3,0.6],0.45)	([0.6,0.8],0.7)	([0.8,1.0],0.9)
r_4	([0,0.3],0.15)	([0.3,0.6],0.45)	([0.6,0.8],0.7)	([0.8,1.0],0.9)
r_5	([0,0.13],0.07)	([0.13,0.33],0.23)	([0.33,0.67],0.5)	([0.67,1.0],0.83)
r_6	([0,0.11],0.06)	([0.11,0.33],0.22)	([0.33,0.67],0.5)	([0.67,1.0],0.83)
r_7	([0,0.25],0.13)	([0.25,0.5],0.38)	([0.5,0.75],0.63)	([0.75,1.0],0.88)
r_8	([0,0.3],0.15)	([0.3,0.6],0.45)	([0.6,0.8],0.7)	([0.8,1.0],0.9)
r_9	([0,0.3],0.15)	([0.3,0.6],0.45)	([0.6,0.8],0.7)	([0.8,1.0],0.9)

Table 3
Data of eight slope samples Q_s (s = 1, 2, …, 8).

r_i	Q_1	Q_2	Q_3	Q_4
r_1	([2,4],3)	([1,3],2)	([3,5],4)	([1,3],2)
r_2	([7,9],8)	([8,10],9)	([3,5],4)	([0,2],1)
r_3	([6,8],7)	([6,8],7)	([7,9],8)	([6,8],7)
r_4	([7,9],8)	([8,10],9)	([3,5],4)	([0,2],1)
r_5	([6,15],10)	([0,46.5],23)	([2,10],6)	([0,32],16)
r_6	([65,85],75)	([20,64],42)	([15,60],38)	([40,75],58)
r_7	([111,210],163)	([107,173],140)	([134,146],140)	([111,185],145)
r_8	([7,9],8)	([7,9],8)	([4,6]5)	([1,3],2)
r_9	([7,9],8)	([7,9],8)	([3,5],4)	([1,3],2)
r_i	Q_5	Q_6	Q_7	Q_8
r_1	([8,10],9)	([5,7],6)	([8,10],9)	([8,10],9)
r_2	([6,8],7)	([7,9],8)	([7,9],8)	([8,10],9)
r_3	([6,8],7)	([6,8],7)	([5,7],6)	([5,7],6)
r_4	([6,8],7)	([7,9],8)	([7,9],8)	([8,10],9)
r_5	([2,28],15)	([11,14],12)	([8,65],57)	([35,85],60)
r_6	([60,75],68)	([25,45],35)	([60,85],73)	([55,70],63)
r_7	([130,160],145)	([107,173],140)	([130,190],160)	([71,217],144)
r_8	([6,8],7)	([7,9],8)	([7,9],8)	([8,10],9)
r_9	([6,8],7)	([7,9],8)	([8,10],9)	([8,10],9)

Table 4
Normalized data (FCSs) of eight slope samples Q_s (s = 1, 2, …, 8).

r_i	Q_1	Q_2	Q_3	Q_4
r_1	([0.2,0.4],0.3)	([0.1,0.3],0.2)	([0.3,0.5],0.4)	([0.1,0.3],0.2)
r_2	([0.7,0.9],0.8)	([0.8,1.0],0.9)	([0.3,0.5],0.4)	([0,0.2],0.1)
r_3	([0.6,0.8],0.7)	([0.6,0.8],0.7)	([0.7,0.9],0.8)	([0.6,0.8],0.7)
r_4	([0.7,0.9],0.8)	([0.8,1.0],0.9)	([0.3,0.5],0.4)	([0.0,0.2],0.1)
r_5	([0.04,0.1],0.07)	([0,0.31],0.15)	([0.01,0.07],0.04)	([0,0.21],0.11)
r_6	([0.72,0.94],0.83)	([0.22,0.71],0.47)	([0.17,0.67],0.42)	([0.44,0.83],0.64)
r_7	([0.56,1.05],0.82)	([0.54,0.87],0.7)	([0.67,0.73],0.7)	(0.56,0.93],0.73)
r_8	([0.7,0.9],0.8)	([0.7,0.9],0.8)	([0.4,0.6]0.5)	([0.1,0.3],0.2)
r_9	([0.7,0.9],0.8)	([0.7,0.9],0.8)	([0.3,0.5],0.4)	([0.1,0.3],0.2)
r_i	Q_5	Q_6	Q_7	Q_8
r_1	([0.8,1.0],0.9)	([0.5,0.7],0.6)	([0.8,1.0],0.9)	([0.8,1.0],0.9)
r_2	([0.6,0.8],0.7)	([0.7,0.9],0.8)	([0.7,0.9],0.8)	([0.8,1.0],0.9)
r_3	([0.6,0.8],0.7)	([0.6,0.8],0.7)	([0.5,0.7],0.6)	([0.5,0.7],0.6)
r_4	([0.6,0.8],0.7)	([0.7,0.9],0.8)	([0.7,0.9],0.8)	([0.8,1.0],0.9)
r_5	([0.01,0.19],0.1)	([0.07,0.09],0.08)	([0.05,0.43],0.38)	([0.23,0.57],0.4)
r_6	([0.67,0.83],0.76)	([0.28,0.5],0.39)	([0.67,0.94],0.81)	([0.61,0.78],0.7)
r_7	([0.65,0.8],0.73)	([0.54,0.87],0.7)	([0.65,0.95,0.8)	([0.36,1.09],0.72)
r_8	([0.6,0.8],0.7)	([0.7,0.9],0.8)	([0.7,0.9],0.8)	([0.8,1.0],0.9)
r_9	([0.6,0.8],0.7)	([0.7,0.9],0.8)	([0.8,1.0],0.9)	([0.8,1.0],0.9)

Then, we calculate the tangent similarity measures between the FCS of the sample Q_s (s = 1, 2, …, 8) and the FCSs of the four stability evaluation grades S_j (j = 1,2,3,4) by using Equation (2), where ω_i (i = 1, 2, …, 9) takes 1/9. Thus, the measured values of $T_s\left(Q_s, S_j\right)$ (s = 1, 2, ..., 8; j

=1,2,3,4) and ranking are given in Table 5. The evaluated grade S_{j*} is obtained by $j^* = \arg\max_{1 \le j \le 4}\{T_s(Q_s, S_j)\}$.

Table 5
Measure values of $T_s(Q_s, S_j)$ (s = 1,2,...,8; j =1,2,3,4) and ranking.

Q_s	$T_s(Q_s, S_j)$	Ranking	Evaluated result	Actual result
Q_1	0.5738,0.7355,0.8469,0.8154	$T_3>T_4>T_2>T_1$	Slight instability (S_3)	S_3
Q_2	0.6015,0.7584,0.8617,0.7942	$T_3>T_4>T_2>T_1$	Slight instability (S_3)	S_3
Q_3	0.7382,0.8851,0.8138,0.6560	$T_2>T_3>T_1>T_4$	Slight stability (S_2)	S_2
Q_4	0.8202,0.7719,0.7046,0.5558	$T_1>T_2>T_3>T_4$	Stability (S_1)	S_1
Q_5	0.5702,0.7614,0.9163,0.8269	$T_3>T_4>T_2>T_1$	Slight instability (S_3)	S_3
Q_6	0.5966,0.7614,0.9032,0.7993	$T_3>T_4>T_2>T_1$	Slight instability (S_3)	S_3
Q_7	0.5028,0.7154,0.8714,0.8923	$T_4>T_3>T_2>T_1$	Instability (S_4)	S_4
Q_8	0.4842,0.7012,0.8634,0.9063	$T_4>T_3>T_2>T_1$	Instability (S_4)	S_4

For convenient comparison, the actual grades in Table 5 were given by the actual cases in [13].

According to the results of Table 5, all the evaluation results of slope stability grades are the same as the actual results in [13], which show the feasibility of the proposed evaluation method in FCS setting. Compared with existing evaluation approaches [6,13], the proposed evaluation method not only is simpler than existing evaluation approaches [6,13] but also can deal with the evaluation problems of slope stability with FCS information.

4. Conclusion

Due to no study of FCSs in engineering areas, this study proposed the tangent similarity measure of FCSs and developed its evaluation method of slope stability. Since FCS can effectively express the hybrid information of both an IVFN and a fuzzy value in the slope stability analysis, the proposed evaluation method provides a new effective way for the slope stability analysis under uncertain and certain environments. Then, eight slope samples are provided as the actual cases to show that the eight evaluation results of slope stability based on the proposed similarity measure of FCSs are in accordance with the actual results of the eight actual cases in [13], which indicate the effectiveness and applicability of the proposed method under FCS environment. However, existing common methods cannot represent and handle the evaluation problems of slope stability with FCS information, while the proposed new method is simpler than existing common methods and can handle the evaluation problems of slope stability with FCS information, which also indicate the main advantages in this study. In future work, we need to propose more measure algorithms of FCSs and to apply them to the clustering analysis and assessment of slope stability in FCS environments.

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Author contributions

J. Ye presented the tangent similarity measure of FCSs and the evaluation method of slope stability; J.M. Wang and W.H. Cui carried out the data processing and slope stability analysis of actual cases; we wrote the paper together.

Conflict of interest

The author declares that this paper has no conflict of interest.

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