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Scale Effect and Anisotropic Analysis of Rock Joint Roughness Coefficient Neutrosophic Interval Statistical Numbers Based on Neutrosophic Statistics

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ABSTRACT

In rock mechanics, mechanical properties of rock masses in nature imply complexity and diversity. The shear strength of rock mass is a key factor for affecting the stability of the rock mass. Then, the joint roughness coefficient (JRC) of rock indicates an important parameter in the shear strength and stability analysis of rock mass. Since the nature of the rock mass is indeterminate and incomplete to some extent, we cannot always express rock JRC by a certain/exact number. Therefore, this paper introduces neutrosophic interval statistical numbers (NISNs) based on the concepts of neutrosophic numbers and neutrosophic interval probability to express JRC data of the rock mass in the indeterminate setting. Then we present the calculational method of the neutrosophic average value and standard deviation of NISNs based on neutrosophic statistics. Next, by an actual case, the neutrosophic average value and standard deviation of the rock JRC NISNs are used to analyze the scale effect and anisotropy of the rock body corresponding to different sample lengths and measuring directions. Lastly, the analysis method of the scale effect and anisotropy for JRC NISNs shows its effectiveness and rationality in the actual case study.

1. Introduction

In 1973, Barton [1] first put forward the concept of joint roughness coefficient (JRC) and indicated its important influence on the shear strength of rocks. It is used to express the empirical coefficient of rough surface undulation to indicate the shear strength of the rock mass structural plane [2]. However, mechanical parameters in nature, such as rock JRC, usually contain imprecise, incomplete, and uncertain information, which is difficultly expressed by certain/exact values. Thus, it is obvious that the classical statistical methods cannot be fit for the expression and analysis of the uncertain JRC data. So Ye et al. [3] first introduced the neutrosophic functions of JRC and shear strength for analyzing the scale effect and anisotropy of JRC and the shear strength of rock mass based on neutrosophic/interval functions [4]. Since the neutrosophic number (NN) introduced in [5–7] is very suitable for expressing the hybrid information of both certain and uncertain measuring values, Ye et al. [8] presented the expression and analysis method of JRC based on NN functions. Next, Chen et al. [9] utilized the NNs of JRC to analyze the scale effect and anisotropy of the uncertain JRC data using neutrosophic statistics [7]. Chen et al. [10] further proposed the concepts of neutrosophic interval probability (NIP) and neutrosophic interval statistical numbers (NISNs) based on the concepts of neutrosophic statistics and NNs, and then applied them to the expressions of uncertain JRC data (JRC NISNs). Unfortunately, there is no study on analyzing scale effect and anisotropy of JRC NISNs by using the neutrosophic statistical analysis method of JRC NISNs in existing literature [3–10]. To solve this gap, this study proposes the calculational method of the neutrosophic average value (NAV) and neutrosophic standard deviation (NSD) of NISNs based on neutrosophic statistics for the first time, and then applies them to analyze the scale effect and anisotropy of the rock mass based on the measured data of rock mass in Changshan County, Zhejiang Province, China as an actual case. However, the analysis method of the scale effect and anisotropy for JRC NISNs can provide an effective and reasonable way under uncertain environments.

In order to do work, Section 2 reviews some basic concepts of NN, NIP, and NISN. In Section 3, we present the calculational method of NAV and NSD for NISNs based on neutrosophic statistics. In Section 4, the indeterminate JRC data are represented by NISNs based on an actual case, and then the scale effect and anisotropy of the JRC NISNs are analyzed by using NAVs and NSDs of JRC NISNs. Lastly, conclusions and next work are indicated in Section 5.

2. Basic concepts of NNs and NISNs

2.1. Neutrosophic numbers

The JRC property of rock surface shape is usually uncertain and incomplete in the measuring and data processing process due to a lack of known/exact information in the indeterminate setting. We cannot always express it by a certain/exact value, which limits the possibility of the exact expression and analysis to some extent. Then NN [5–7] can express the problem of partial certainty and/or partial uncertainty because it can be simply expressed as $y = p+qI$ (p and q are any real numbers), where p is the certain part and qI is the uncertain part with its indeterminacy I. Obviously, NN easily represents the certain and/or uncertain information in indeterminate

setting. For instance, $y = 3+2I$ for $I \in [0,1]$ or $I \in [2,3]$ indicates $y \in [3,5]$ or $y \in [7,9]$, which implies a changeable interval number depending on I. Hence, NN show its convenient and flexible advantage in the expression of certain and/or uncertain information [8–10].

2.2. Neutrosophic interval probability

Chen et al. [10] first proposed the concept of NIP to express the NIP of uncertain JRC data in the indeterminate setting.

Set $a = [a^l, a^u]$ as the interval range of measuring data for a group of samples. Chen et al. [10] defined a NIP N and expressed it as $N = \langle[a^l, a^u], (T_p, I_p, F_p)\rangle$, where T_p is its truth probability in the certain interval, I_p is its indeterminate probability in the uncertain interval; and F_p is its falsity probability in the almost impossible/failure interval, for $T_p + I_p + F_p = 1$.

For instance, suppose that there is a group of n samples regarding the same size. Then we calculate the average value m and standard deviation σ of the JRC data obtained from the n samples. Thus, the interval value $a = [a^l, a^u]$ is obtained when a^l is the minimum value (the lower bound) of the measuring data and a^u is the maximum value (the upper bound) of the measuring data. Then, we can define $[m-\sigma, m+\sigma]$ as the confident/truth interval, $[m-3\sigma, m-\sigma) \cup (m+\sigma, m+3\sigma]$ as the uncertain intervals, and $[a^l, m - 3\sigma) \cup (m + 3\sigma, a^u]$ as the remaining/incredible intervals. Hence, we can calculate the truth probability $T_p = n_T/n$ for the frequency n_T in $[m-\sigma, m+\sigma]$, the uncertain probability $I_p = n_I/n$ for the frequency n_I in $[m-3\sigma, m-\sigma) \cup (m+\sigma, m+3\sigma]$, and the falsity probability $F_p = n_F/n$ for the frequency n_F in $[a^l, m-3\sigma) \cup (m+3\sigma, a^u]$.

2.3. Neutrosophic interval statistical numbers

Chen et al. [10] also presented the concept of NISN and used it for the expression of JRC with uncertain information corresponding to the NN and confidence degree.

Chen et al. [10] defined the following confidence degree:

$$c = T_p/\sqrt{T_p^2 + I_p^2 + F_p^2}, \tag{1}$$

and then presented a NISN, which is expressed as

$$S = m + (1 - c)\sigma I \text{ for } I \in [I^l, I^u], \tag{2}$$

where m and σ are the average value and standard deviation of a group of JRC data, respectively.

Firstly, we consider that the indeterminate JRC value is represented by NISN based on an actual example below.

Example 1. In an actual example, we take a group of 290 measured data ($n = 290$) of the sample length $L = 100$cm and measuring direction $0°$, and then express the indeterminate JRC value by NISN. Thus, Table 1 indicates the statistical values of the obtained JRC data.

Table 1
Statistical data of the obtained JRC data for the actual example.

n	m	σ	$[a^l, a^u]$	n_T	n_I	n_F
290	13.20	4.35	[4.63, 25.82]	205	70	15

Hence, the indeterminate JRC value is expressed by NISN below.

From Table 1, we can calculate the truth, indeterminacy, and falsity probabilities in the interval [4.63, 25.82] as follows:

$P_T = 205/290 = 0.71$, $P_I = 70/290 = 0.24$, $P_F = 15/290 = 0.05$.

Then, the NIP of JRC is expressed as $N = <[4.63, 25.82], (0.71, 0.24, 0.05)>$.

By Eq. (1), the confidence degree is

$$c = T_p/\sqrt{T_p^2 + I_p^2 + F_p^2} = \frac{0.71}{\sqrt{0.71^2 + 0.24^2 + 0.05^2}} = 0.9065.$$

By Eq. (2) the NISN of JRC is expressed as

$S = m + (1 - c)\sigma I = 9.8725 + 0.0935 \times 4.35I = 9.8725 + 0.4067I$.

Suppose the indeterminacy I is specified as $I \in [-1, 1]$. Then, there is S = [9.4658, 10.2792].

3. Neutrosophic statistics of NISNs

In this section, we propose the neutrosophic statistical method to calculate the NAV and NSD of NISNs.

For a group of $S_i = m_i + (1 - c_i)\sigma_i I$ for $I \in [I^l, I^u]$ and $i = 1, 2, \ldots, n$, we set $b_i = (1 - c_i)\sigma_i$, then there is $S_i = m_i + b_i I$ for $I \in [I^l, I^u]$. Thus the NAV of S_i is presented as the following formula:

$$S_m = A_m + B_m I \text{ for } I \in [I^l, I^u], \tag{3}$$

where $A_m = \frac{1}{n}\sum_{i=1}^{n} m_i$ and $B_m = \frac{1}{n}\sum_{i=1}^{n} b_i$ for the average value m_i and the indeterminacy coefficient $b_i = (1 - c_i)\sigma_i$.

Then, the NSD of NISNs S_i is proposed as

$$\delta = \sqrt{\frac{1}{n}\sum_{i=1}^{n}(S_m - S_i)^2}, \tag{4}$$

where $S_m - S_i$ is calculated by

$$S_m - S_i = A_m - m_i + (B_m - b_i)I \text{ for } I \in [I^l, I^u], \tag{5}$$

and then there is the following result:

$$(S_m - S_i)^2 = \{min[(A_m + B_m I^l)(m_i + b_i I^l), (A_m + B_m I^l)(m_i + b_i I^u), (A_m + B_m I^u)(m_i + b_i I^l), (A_m + B_m I^u)(m_i + b_i I^u)], max[(A_m + B_m I^l)(m_i + b_i I^l), (A_m + B_m I^l)(m_i + b_i I^u), (A_m + B_m I^u)(m_i + b_i I^l), (A_m + B_m I^u)(m_i + b_i I^u)]\}$$

for $I \in [I^l, I^u]$. $\tag{6}$

4. Scale effect and anisotropic analysis of rock JRC NISNs

In order to reflect the scale effect and anisotropy of rock JRC data with uncertain information in different sample sizes and different measuring directions, this section expresses and analyzes

JRC NISNs from different sample lengths (from 10cm to 100cm) and different measuring directions (from 0° to 345°). Thus the JRC NISNs can be yielded by using Eqs. (1) and (2) from the obtained JRC data, and then analyzed by the proposed neutrosophic statistical method.

4.1. Scale effect of JRC NISNs regarding various sample lengths

In each sample length L_j (cm) ($j = 1, 2, …, 10$), there is a group of 24 JRC NISNs $S_{ji} = m_{ji} + b_{ji}I$ for $I \in [-1, 1]$ ($j = 1, 2, …, 10; i = 1, 2, …, 24$) in 24 measured directions, where $b_{ji} = (1 - c_{ji})\sigma_{ji}$. Thus, by Eqs. (3)-(6) we can get the NAVs $S_{mj} = A_{mj} + B_{mj}I$ for $I \in [-1, 1]$, where $A_{mj} = \frac{1}{24}\sum_{i=1}^{24} m_{ji}$ and $B_{mj} = \frac{1}{24}\sum_{i=1}^{24} b_{ji}$, and the NSDs $\delta_j = \sqrt{\frac{1}{24}\sum_{i=1}^{24}(S_{mji} - S_{ji})^2}$ ($j = 1, 2, …, 10$) for the 24 NISNs of JRC, and then the detailed statistical results are shown in Table 2. The NAVs S_{mj} and NSDs δ_j ($j = 1, 2, …, 10$) regarding the different sample lengths of L_j (cm) ($j = 1, 2, …, 10$) are indicated in Figures 1 and 2.

Table 2
Neutrosophic statistical results of JRC NISNs in different sample lengths for $I \in [-1, 1]$.

| L_j (cm)	A_{mj}	B_{mj}	S_{mj}	δ_j
10	9.8725	0.9065	9.8725+0.4067I	[1.2765,1.6548]
20	9.3768	0.8653	9.3768+0.4276I	[1.1056,1.3976]
30	9.1636	0.9354	9.1636+0.4316I	[0.9783,1.1976]
40	8.7572	0.8569	8.7572+0.4637 I	[0.8759,1.0862]
50	8.6584	0.8593	8.6584+0.3985I	[0.8237,0.9856]
60	8.4975	0.8368	8.4975+0.3975I	[0.7927,0.9286]
70	8.3975	0.9156	8.3975+0.4028I	[0.7659,0.8858]
80	7.9763	0.8563	7.9763+0.4217I	[0.7376, 0.8525]
90	7.5694	0.8529	7.5693+0.4816I	[0.7068,0.8158]
100	7.3539	0.8625	7.3539+0.3978I	[0.6895,0.7913]
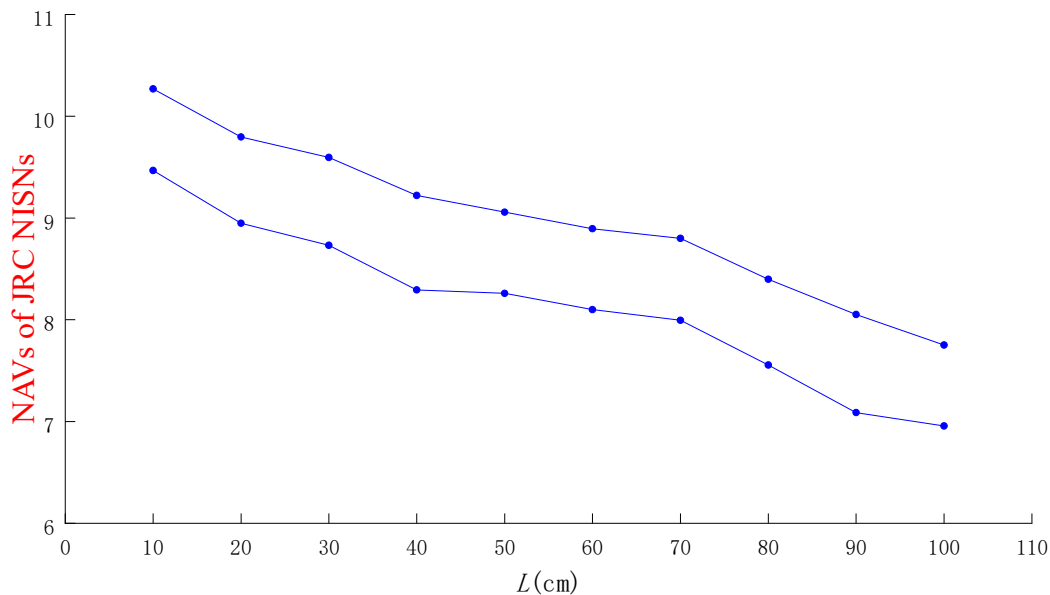
Fig. 1. NAVs of JRC NISNs in different sample lengths for $I \in [-1, 1]$.

As it can be seen from Figure 1, the NAVs of JRC NISNs indicate a decreasing trend with increasing sample length, which is in accordance with the previous results [8–10], and then show their scale effect in different sample lengths. Hence, the bigger the sample length L is, the smaller the NAV of JRC NISNs is, i.e., the smaller the scale effect is. It is clear that the NAVs of JRC NISNs can describe the scale effect in different sample lengths.

Since NSD can reflect the dispersion degree of the NAVs of JRC NISNs, the characteristics of the scale effect in different sample lengths can be indicated by the NSDs of JRC NISNs. In Figure 2, as the sample length increases, the NSDs of JRC present a decreasing tendency and narrow the deviation range, which shows their scale effect nature in different sample lengths. Obviously, the bigger the sample length L is, the smaller the NSD value/range of JRC NISNs is, i.e., the smaller the dispersion degree of the NAVs of JRC NISNs is. It is clear that the NSDs of JRC NISNs can also describe the scale effect in different sample lengths. What's more, the NSD value and range of the JRC NISNs may demonstrate the stable tendency to some extent when the sample length is large enough.

From the above analyses, we can see that the NAVs and NSDs of JRC NISNs can indicate the scale effect of the JRC NISNs in different sample lengths.
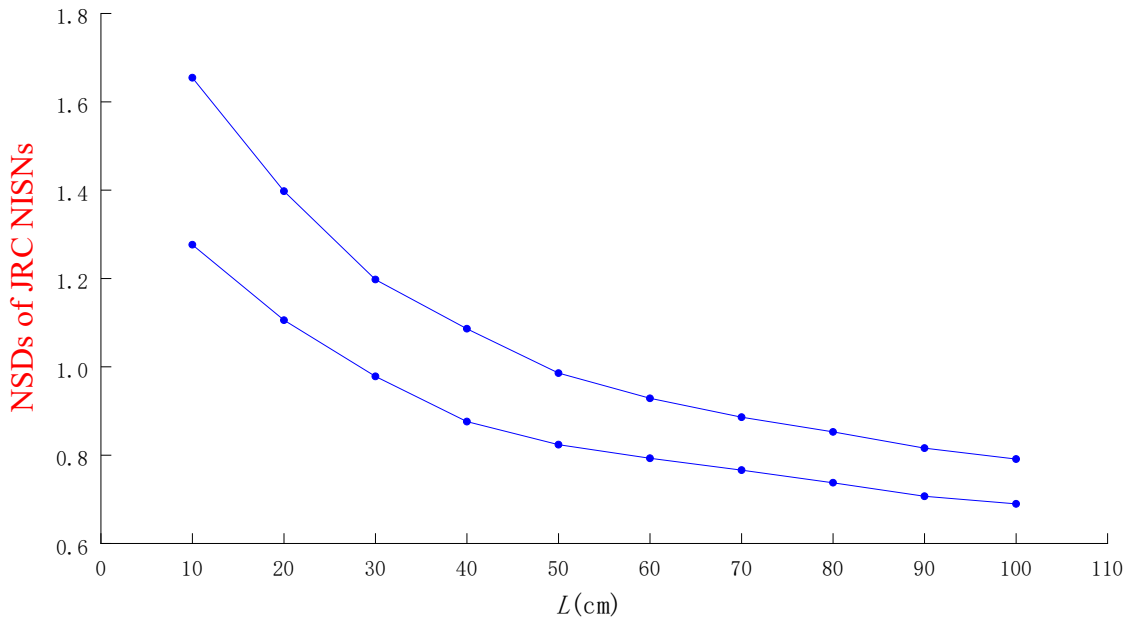

Fig. 2. NSDs of JRC NISNs in different sample lengths for $I \in [-1, 1]$.

4.2. Anisotropy of JRC NISNs regarding different measuring directions

In each measuring direction θ_i (°) ($i = 1, 2, …, 24$) there is a group of 10 NISNs of JRC $S_{ji} = m_{ji} + b_{ji}I$ for $I \in [-1, 1]$ ($j = 1, 2, …, 10; i = 1, 2, …, 24$) in 10 sample lengths, where $b_{ji} = (1 - c_{ji})\sigma_{ji}$. Thus, by Eqs. (3)-(6) we can get the NAVs $S_{mi} = A_{mi} + B_{mi}I$ for $I \in [-1,1]$, where $A_{mi} = \frac{1}{10}\sum_{j=1}^{10} m_{ji}$ and $B_{mi} = \frac{1}{10}\sum_{j=1}^{10} b_{ji}$, and the NSDs $\delta_i = \sqrt{\frac{1}{10}\sum_{j=1}^{10}(S_{mji} - S_{ji})^2}$ ($i = 1, 2, …, 24$) for the 10 NISNs of JRC, and then the detailed statistical results are shown in Table 3. The NAVs S_{mi}

and NSDs δ_i (i = 1, 2, …, 24) regarding the different measuring directions of θ_i (°) (i = 1, 2, …, 24) are indicated in Figures 3 and 4.

Table 3
Neutrosophic statistical results of JRC NISNs in different measuring directions for $I \in [-1, 1]$

| θ_i	A_{mi}	B_{mi}	S_{mi}	δ_i
0°	8.2857	0.1137	8.2857+0.3083I	[0.3652,0.8562]
15°	8.5632	0.1248	8.5632+0.3258I	[0.4026,0.7856]
30°	8.3628	0.1211	8.3628+0.4027I	[0.5142,0.8756]
45°	7.9768	0.1579	7.9768+0.3973I	[0.4982,0.7063]
60°	7.3762	0.3786	7.3762+0.4126I	[0.5317,0.9157]
75°	7.8549	0.2747	7.8549+0.3347I	[0.3876,0.7964]
90°	7.8361	0.3048	8.2651+0.4021I	[0.1785,0.5742]
105°	8.7693	0.2368	7.9693+0.3964I	[0.2867,0.7652]
120°	7.5736	0.1248	7.8736+0.3351I	[0.4875,0.8745]
135°	6.7427	0.2371	8.1027+0.4147I	[0.6725,0.9652]
150°	7.7542	0.1348	7.7542+0.3782I	[0.7821,1.0476]
165°	7.9654	0.1651	7.9654+0.4624I	[0.6756,0.9752]
180°	8.5218	0.1737	8.5218+0.3853I	[0.5672,0.8527]
195°	8.7652	0.2374	8.7652+0.3972I	[0.3786,0.7635]
210°	8.9217	0.2149	8.6217+0.4371I	[0.1045,0.5752]
225°	7.9762	0.1368	7.9762+0.4214I	[0.3028,0.8368]
240°	7.6582	0.2482	7.6582+0.3958I	[0.5162,0.8263]
255°	7.3813	0.0284	7.3813+0.4083I	[0.7251,1.0517]
270°	6.8724	0.0482	6.8724+0.4524I	[0.8162,1.2635]
285°	6.7632	0.2069	6.9632+0.3895I	[0.6328,0.9732]
300°	7.6652	0.1372	7.8652+0.4628I	[0.7231,1.2742]
315°	7.8653	0.1269	7.8653+0.3731I	[0.4823,0.8136]
330°	7.8972	0.2039	7.8972+0.3961I	[0.6731,0.9851]
345°	8.6328	0.1049	8.6328+0.4951I	[0.4719,0.8937]
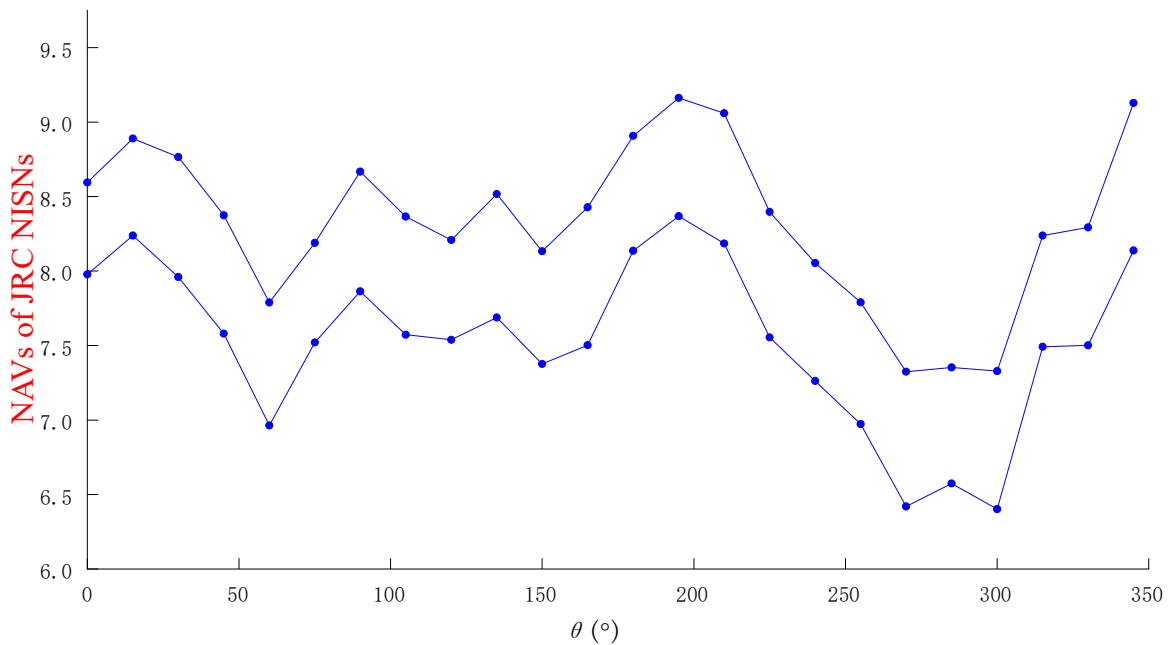
Fig. 3. NAVs of JRC NISNs in different measuring directions for $I \in [-1, 1]$.

According to Figure 3, the NAVs of JRC NISNs in different measuring directions are very different in every orientation. The changing curves show some volatility, which looks somewhat like the curves of trigonometric functions, and the anisotropy of JRC NISNs in different measuring directions.
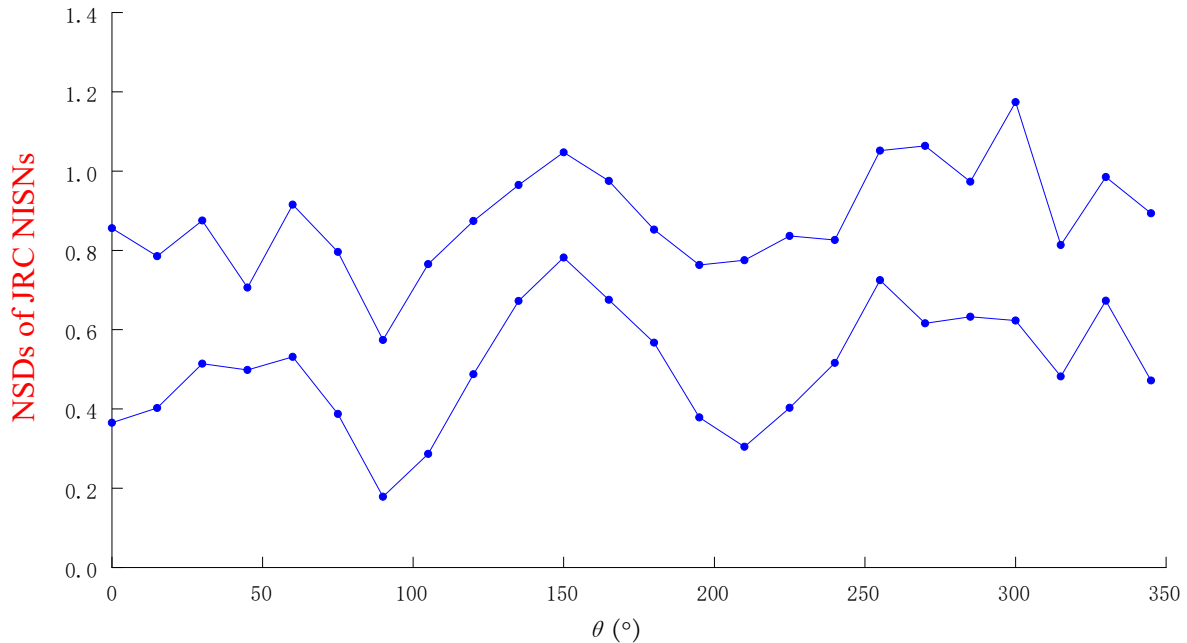

Fig. 4. NSDs of JRC NISNs in different measuring directions for $I \in [-1, 1]$.

In Figure 4, the NSDs of JRC NISNs in different measuring directions are very different in every orientation, which shows different dispersion degrees of JRC NISNs in every orientation. Then, the changing curves also show some volatility, which looks somewhat like the curves of trigonometric functions, and the anisotropy of JRC NISNs. Obviously, the NSDs of JRC NISNs also fluctuate in different measuring directions, which can also reflect the anisotropic nature to some extent.

From the above analyses, we can see that the NAVs and NSDs of JRC NISNs can also indicate the anisotropic nature of the JRC NISNs in different measuring directions.

Therefore, the neutrosophic statistical analysis method of JRC NISNs contains much more information and then is more reasonable and effective than the existing statistical analysis method of JRC NNs in the reference [9] without considering its neutrosophic interval probability and a neutrosophic confident degree in indeterminate situations. Furthermore, the neutrosophic statistical analysis method of JRC NISNs further extends our previous study in the reference [10] that did not use the neutrosophic statistical analysis method in this study for the scale effect and anisotropic analysis of JRC NISNs.

5. Conclusion

Since NISNs can effectively express JRC data with uncertain information from the viewpoint of the neutrosophic probability and statistics, for the first time this study proposed the neutrosophic statistical method of NISNs to calculate the NAVs and NSDs of NISNs. Then, the NAVs and NSDs of JRC-NISNs were utilized to analyze the scale effect and anisotropy of JRC NISNs in different sample lengths and measuring directions by an actual case study. Main advantages of this study are that: (1) the NAVs and NSDs of JRC NISNs contain much more useful information to overcome the insufficiency of losing useful information produced in existing expression and analysis methods of uncertain JRC data, (2) the scale effect and anisotropy of JRC NISNs analyzed by both the NAVs and NSDs of JRC NISNs are more reasonable and more effective than those methods expressed and analyzed by JRC NNs [8,9], and (3) the scale effect and anisotropy of JRC-NISNs analyzed by both the NAVs and NSDs of JRC NISNs can extend existing expression and analysis methods of JRC NNs [8,9] and overcome the insufficiency without the neutrosophic statistical analysis of JRC NISNs in [10]. Obviously, the scale effect and anisotropy of JRC NISNs analyzed by using the neutrosophic statistical method of JRC NISNs are effective and reasonable in this case study and extend existing various analysis methods. The next work will extend the neutrosophic interval probability and statistics to the expression and analysis of the shear strength of rock mass in rock mechanics.

Acknowledgment

Author contributions

J. Ye proposed the NISN statistics and analysis method; W.Z. Jiang and W.H. Cui carried out the measured data processing and calculations and utilized them to analyze the scale effect and anisotropy of NISNs of rock JRC in rock mechanics; we wrote the paper together.

Conflicts of interest

The authors declare no conflicts of interest.

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