

Contents lists available at SCCE

Journal of Soft Computing in Civil Engineering



Journal homepage: www.jsoftcivil.com

Predicting the Earthquake Magnitude along Zagros Fault Using Time Series and Ensemble Model

A. Shishegaran^{1*}, H. Taghavizade², A. Bigdeli³, A. Shishegaran⁴

1. Ph.D. Candidate, Environmental Engineering, School of Civil Engineering, Iran University of Science and Technology, Tehran, Iran

2. Research Assistant, Geotechnical Earthquake Engineering, International Institute of Earthquake Engineering and Seismology, Tehran, Iran

3. Master of Science Student, Structural Engineering, School of Civil Engineering, Iran University of Science and Technology, Tehran, Iran

4. Master of Science Student, Structural Engineering, School of Civil Engineering, Islamic Azad University, Tehran, Iran

Corresponding author: aydin_shishegaran@civileng.iust.ac.ir

doi https://doi.org/10.22115/SCCE.2020.213197.1152

ARTICLE INFO

Article history: Received: 25 December 2019 Revised: 14 January 2020 Accepted: 20 January 2020

Keywords: Earthquake magnitude; Time series; Statistical parameters; Prediction; Ensemble technique.

ABSTRACT

Predicting the earthquake magnitude is a complex problem, which has been carried out in recent years. The machine learning, geophysical, and regression methods were used to predict earthquake magnitude in literature. Iran is located in a highly seismically active area; thus, earthquake prediction is considered as a great demand there. In this study, two time series algorithms are utilized to predict the magnitude of the earthquake based on previous earthquakes. These models are autoregressive conditional heteroscedasticity (GARCH), autoregressive integrated moving average (ARIMA), and the combination of ARIMA and GARCH by multiple linear regression (MLR) technique (model 3). The 9017 events are used to train and predict earthquake magnitude. On the other hand, 6188 events are applied for training models, and then 2829 events are utilized for testing it. The statistical parameters, such as correlation coefficient, root mean square error (RMSE), normalized square error (NMSE), and fractional bias, are calculated to evaluate the accuracy of each model. The results demonstrate that the ARIMA and model 3 can predict future earthquake magnitude better than other models.

How to cite this article: Shishegaran A, Taghavizade H, Bigdeli A, Shishegaran A. Predicting the earthquake magnitude along zagros fault using time series and ensemble model. J Soft Comput Civ Eng 2019;3(4):67–77. https://doi.org/10.22115/scce.2020.213197.1152.



1. Introduction

Natural Hazards, such as earthquakes, have caused great human and economic losses; however, a widespread demand is emerged during recent years for predicting such events in order to prepare and diminish their effects on infrastructures properly. Literally, the concept of natural risk can be interpreted as a relative consciousness of society; nevertheless, there is no universally accepted definition for this issue [1]. Nonetheless, this issue can be technically defined as an extent to which the probability of the hazardous event and its potential implications are combined.

The occurrence of natural hazards can be referred to when such a likely disaster turns in to a real one causing human and economic losses. For its severe effects, earthquake is recognized as the most devastating natural phenomena among others, such as tsunamis, floods, tornadoes, and volcanic eruption [2]. An earthquake occurs suddenly without any definite sign leading to loss of lives or injuries, buildings and infrastructure devastations, social and economic losses, and even environmental pollution [3]. Besides, numerous regions having a highly dense population are located within seismic areas. In addition to ground shaking, an earthquake can produce corresponding effects, such as liquefaction [4], landslide [5], and tsunamis [6].

Seismic risk can be defined as the composition of seismic vulnerability and seismic hazard [7]. In one aspect, seismic hazard describes a potentially seismic event causing damages and losses. In the other aspect, it is the potential vulnerability, which can show the extent of the destructive effect of hazard.

Using the big datasets can assist in comprehending regression prediction models better; therefore, the significant data can improve the models for better prediction [8]. Increasing our knowledge in the earthquake can assist in improving the risk management and seismic hazard [9].

The present investigation forecasts the earthquake information in one region of Iran where has experienced numerous earthquakes since 2009 and has a large seismic information database. As large population growth has been observed in this area. Moreover, the earthquake is considered to be a major threat to urbanization and for the people who lived there. The time series models, such as ARIMA, GARCH, the combination of GARCH and ARIMA, are used in this investigation to predict the magnitude of possible earthquakes in this area. Although datasets used in previous studies were limited to serval Megabyte, Asencio-Cortes et al. employed 1GB datasets to predict an earthquake in California [10,11]. In addition, there are several useful studies, which focused on the regression and neural network (NN) models for predicting outputs [12–22]. They used several regressor models to predict an earthquake in the future for seven days. The novelty of this study is related to the prediction of earthquake magnitude by time series and the ensemble model, which is developed by two time series models.

2. Methodology

Three models are applied to achieve the aim of the earthquake magnitude prediction in the Zagros fault line, which is larger than $2.5(M \ge 2.5)$. The earthquake dataset is acquired from the

seismological networks of the Institute of Geophysics at the University of Tehran (IGTU). Table 1 demonstrates the most important features of the events catalog. According to Table 1, these data include 9017 earthquake events that occurred at three states of Iran during the time period from January 2009 to November 2018 with a magnitude of 2.5 or higher.

Primary specifications of the original dataset used in the current study.						
Zone	Ilam, Khuzestan and Bushehr states (Iran)					
Source	Iranian Seismological Center (IGTU)					
Period	2009-2018					
Total Events	9017					
	Zone Source Period					

According to the catalog of the earthquake, statistical features of target variables, which are used for predicting earthquake magnitude, are presented in Table 2. In this dataset, the minimum longitude and latitude are 45.8, 28.47, respectively, and the maximum longitude and latitude are 34.5 and 56.3, respectively. Specially, these longitude and latitude are referred to as west, southwest, south, and center of Iran. Figure 1 shows the regions, which their datasets are used to study in this investigation.

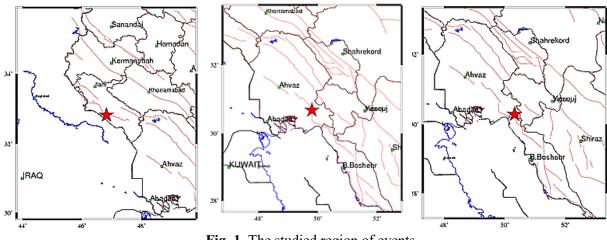


Fig. 1. The studied region of events.

Table 2	
Statistical characteristics of the analytical dataset used for the present regression study	<i>'</i> .

Variable	Min	Max	Mean	Median	Variance
Magnitude	2.5	6.2	2.966	2.8	0.222
Longitude	28.47	34.5	32.01	32.31	1.72
Latitude	45.8	56.3	49.95	49.43	7.856

In the first model, for predicting the next earthquake magnitude, autoregressive conditional heteroscedasticity (GARCH) with mean offset is generalized. The second model is the autoregressive integrated moving average (ARIMA). The third model is the combination of ARIMA and GARCH by multiple linear regression (MLR) technique. In other words, the outputs of ARIMA and GARCH models are imported as the inputs in the MLR model. The value of

earthquake magnitude for a time period between 2009 and 2016 has been imported as input parameters to these models for training the future magnitude. After the training, the calibrated models are achieved, and then these models are employed to predict the earthquake information during the years 2017-2018. Finally, the predicted values of earthquake magnitude accounted as outputs of models are assessed by comparing them with earthquake magnitudes recorded from 2017 to November 2018. The statistical parameters, such as correlation coefficient, root mean square error (RMSE), normal mean square error (NMSE), and fractional bias, are calculated for evaluating the accuracy of the models.

3. Results and discussion

As above-mentioned, the data for a period of 2009-2016 have been applied as input for training the models. Afterward, for a period of 2017-2018, the predicted values are calculated by the calibrated models.

The correlation coefficient between current earthquake magnitude and previous, the second pervious, the third previous, and the fourth previous earthquake magnitudes were evaluated, as shown in Table 3.

Table. 3

Correlation between earthquake data with previous four data's earthquake for magnitude, longitude, latitude and time range of earthquake occurrence.

S.N	variable	correlation between earthquake data and pervious earthquake data	correlation between earthquake data and second previous earthquake data	correlation between earthquake data and second previous earthquake data	correlation between earthquake data and second previous earthquake data
1	Magnitude	0.101	0.060	0.058	0.033

According to the result of Table 3, the model ARIMA (1,1,1) and the GARCH (1, 1) model with mean offset 1 are chosen in this study.

3.1. Prediction models

The GARCH model having mean offset is utilized by the autoregressive moving average model to assume for the error variance, which the mean offset appears in the output as an additional parameter to be estimated or otherwise specified. As above-mentioned, the GARCH (1, 1) model with mean offset 1 is utilized for calculating the prediction of earthquake magnitude in Zagrous faults. The Box-Jenkins methodology refers to a set of procedures for identifying and estimating time series models within the class of ARIMA models [23]. Among different ARIMA models, ARIMA (1,1,1) has been chosen. The third model is the combination of two time series models, which outputs of models 1 and 2 are imported as the input variables in the third model.

3.2. Prediction of magnitude

Magnitude is the scale to describe the overall strength or size of the earthquake. Fig 2 shows the comparison of the predicted magnitudes and the recorded magnitudes for each method. Fig 2 (a)

shows that the results of the predicted magnitude by GARCH are not acceptable. As shown in fig 2. (b), the outputs of ARIMA demonstrate that the predicted magnitudes are close to the recorded values; hence, the ARIMA model can predict acceptable values. According to fig. 2 (c), the predicted values of model 3 are better than model 2; therefore, it seems that model 3 can predict magnitude better than model 2.

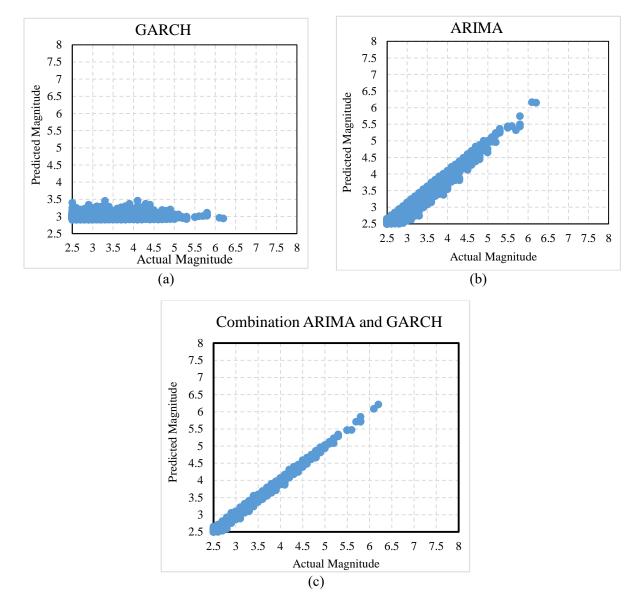


Fig. 2. Comparison of actual and predicted magnitude values for each regression model.

Table 4 shows the statistical parameters that are calculated by using the predicted and recorded magnitude. These parameters can evaluate the accuracy of models for predicting magnitude. According to the results of fractional bias, all models are under-predicting. The correlation coefficient of GARCH is smaller than 0.005, and the value shows that model 1 cannot predict earthquake magnitude; however, the correlation coefficient of ARIMA and model 3 is 0.9852 and 0.9946, respectively. As a result, models 2 and 3 demonstrate acceptable results. The RMSE and

NMSE of models 2 and 3 are obtained 0.0565, 0.0186, and 0.0339, 0.0116, respectively; therefore, model 3 shows better results in comparison to model 2 with respect to the RMSE and NMSE. In addition, model 3 performs better results than model 2 with respect to the correlation coefficient. These parameters show that the models 2 and 3 have been verified and performed to be acceptable in predicting magnitude, although Model 3 can predict earthquake magnitude more accurate than model 2. According to the comparison between the predicted and actual magnitude by Fig. 2 and statistical parameters, the results of model 3 are better than model 2.

Com	Comparison of predicted and monitored magnitude values in years 2009-2016 and years 2017 and 2018.								
S.N	Regression - method	2009-2016			2017-2018				
		Correlation Coefficient	RMSE	NMSE	Fractional Bias	Correlation Coefficient	RMSE	NMSE	Fractional Bias
1	GARCH	0.0182	0.4730	0.8544	-2.43E-08	0.0019	0.4612	1.3647	1.21E-07
2	ARIMA	0.9553	0.1009	0.0248	-3.87E-08	0.9852	0.0565	0.0186	1.86E-07
3	The ensemble model	0.9925	0.0413	0.0106	-2.98E-09	0.9946	0.0339	0.0116	1.44E-08

To develop model 3, the outputs of models 1 and 2 are required; therefore, equations of models 1, 2, and 3 are important to predict the magnitude of the earthquake. The equations (1), (2), and (3) are presented to predict the values of magnitude, which referred to models 1, 2, and 3, respectively. They are formulated as below:

$$M_{P} = 2.09236 + 0.39546 y_{t-1} + 0.09272 \varepsilon_{t-1}$$
⁽¹⁾

$$M_{P} = -8.6507 \times 10^{-6} + \alpha_{1} + 0.07257\alpha_{t-1} - 0.98704\varepsilon_{t-1}$$
⁽²⁾

$$M_{P} = 0.28281G + 0.9976A + 0.20481 \tag{3}$$

Where ε_t and μ are typically assumed to be "white noise" and "mean offset"; i.e., it is identically and independently distributed with a common mean 0 and common variance σ^2 across all observations. y_{t-1} and α_{t-1} are the last magnitudes. G and A are defined as outputs of models 1 and 2, respectively.

The results of ARIMA demonstrate acceptable prediction of magnitude and duration of the earthquake. The ARIMA minimizes the error of the prediction model, although the GARCH minimizes the variance of the prediction model. Based on the ability of the GARCH and ARIMA models, the combination of ARIMA and GARCH by the MLR can assist in obtaining a better equation in comparison to the ARIMA or GARCH.

4. Conclusions

The main aim of this study is related to develop the time series models for predicting the earthquake magnitude. Earthquake events along Zagrous fault from 2009 to 2018, which their

Table. 4

magnitude is more than 2.5, have been employed in this study. The earthquake dataset was acquired from the seismological networks of the Institute of Geophysics at the University of Tehran (IGTU). Two time series and the ensemble model have been utilized to predict the future earthquake magnitude. The results of this study are summarized as follows:

- The results demonstrate that ARIMA is an acceptable model to predict the earthquake magnitude.
- Model 3, which is the combination of ARIMA and GRCH, can predict the earthquake magnitude better than ARIMA. This issue demonstrates that the model 3 algorithm is stronger to predict the earthquake magnitude in Zagrous fault.
- Any physics-related problem is not applied to the models to predict earthquake magnitude. These proposed models are statistical models, and they are based on time series formulation.
- The time series models can be combined with deterministic models such as Gaussian plume or Eulerian models, which perhaps can improve the models.

Appendix A

Model 1: GARCH

The first model is the GARCH method with mean offset. The GARCH is achieved from the autoregressive conditional heteroscedasticity (ARCH). On the other hand, the GARCH was created by developing the ARCH method. The ARCH method is the statistical method, which uses time-series data. This method describes the variance of the current error term or innovation as an equation of the actual sizes of previous time periods error term. The variance is related to the squares of the previous innovation in this method. The GARCH (p, q) model is calculated by the following equations. P is the order of the arch term e^2 . μ is the mean of offset in the GARCH model. The mean offset appears in the output as an additional parameter to be estimated, or otherwise specified.

$$y_t = x_t b + \mu + \varepsilon_t \tag{A1}$$

$$\varepsilon_t / \psi_{t-1} \sim N(0, \sigma_t^2) \tag{A2}$$

$$\sigma_{t}^{2} = \omega + \alpha_{1}\varepsilon_{t-1}^{2} + \dots + \alpha_{q}\varepsilon_{q-1}^{2} + \beta_{1}\sigma_{t-1}^{2} + \dots + \beta_{p}\sigma_{p-1}^{2} = \omega + \sum_{i=1}^{q}\alpha_{i}\varepsilon_{t-i}^{2} + \sum_{i=1}^{q}\beta_{i}\sigma_{t-i}^{2}$$
(A3)

Where $\alpha 0$, $\alpha t > 0$, $(\beta 1 + \alpha 1) < 1$.

The lag length p of a GARCH (p, q) process is established by three steps, in which the first one is to estimate the best fitting AR (q) model, the second one is to compute and plot the auto correlations of $\varepsilon 2$, and the last step is the asymptotic process. The asymptotic used for large samples standard deviation of p(i) is $\frac{1}{\sqrt{T}}$ and individual values are larger than the indication of GARCH errors. For estimating the total number of lags, the Ljung-Box test is utilized. This

process is recommended to consider up to values of n. There are different methods to estimate σ_t^2 in the literature. Many authors have chosen models within the GARCH family [24]. The simplest of these models is the GARCH (1, 1) model [25], which is calculated by equation (A3). In addition, the GARCH model has been implemented to estimate the conditional variance. In the present study, the GARCH (1, 1) model has been applied for simplicity, but the other GARCH prediction models could be utilized as well in future studies.

Model 2: ARIMA

Based on the above-mentioned consideration, the second model is ARIMA in the present study. One of the approaches to analyze the time series data is the Box-Jenkins ARIMA method, which includes exploiting the foreseeable behavior from the monitored values [23]. To explain clearly, the ARIMA procedure of order (p, d, q) is demonstrated by equation (A4), which is formulated as:

$$W_{t} = \sum_{i=1}^{p} \phi W_{t-i} + \alpha_{t} - \sum_{j=1}^{q} \theta_{j} \alpha_{t-j}$$
(A4)

In which $W_t = \nabla^d y_t$ and d is the order of differencing, ∇ is defined as the reversed difference operator, p is the order of autoregressive procedure, and q is the order of moving average procedure. ϕ_i and θ_j are defined as ithautoregressive variables, jth moving average variables, respectively. y_t and α_t are the recorded value at time t and the error parameter at time t. In this method, the correlation coefficient between the current earthquake data and the first, the second, the third, and the fourth previous earthquake data are calculated to select p of the ARIMA model.

Model 3: The combination of ARIMA and GARCH with MLR

The first researchers who defined a combination method instead of one single method were, are Bates and Granger [26]. The opinion of combining predictive algorithms is to keep each model property and to get various properties in the dataset. The model 3 is the combination of ARIMA and GARCH with the MLR method. In other words, the outputs of ARIMA and GARCH with mean offset are independent variables, e is an estimated error term, which is calculated from independent random sampling from the normal distribution with mean zero and constant variance. According to the equation (A5), the minimum square error technique can be used to determine the b1, b2, b3. This solution can be generated as $b=[X^TX]^{-1}[X^TY]$. The formula of model 3 is calculated as:

$$Y = b_1 + b_2 X_2 + b_3 X_3 + e \tag{A5}$$

where X^T is the transpose of X.

Appendix B

The statistical parameters that have been presented by Chang and Hanna are used for assessing the models in the present study [27]. Correlation of coefficient (R) is one of the used statistical

parameters, which the strength of the relationship between the predicted and relative actual values determines. In other words, the association between the predicted and observed values is calculated by this parameter that can be calculated by equation (B1). In is clear that the value of the correlation coefficient is between -1 and 1. If the amount of R is determined -1 and 1, there is a perfect relationship between predicted and actual values. In contrast, a correlation with 0 value demonstrates no relationship between the movement of the predicted and actual values.

$$R = \frac{n(\sum C_o C_p) - (\sum C_o)(\sum C_p)}{\sqrt{[n \sum C_o^2 - (\sum C_o)^2][n \sum C_p^2 - (\sum C_p)^2]}}$$
(B1)

In which n, C_o and C_p are defined as number in the given dataset, the observed earthquake magnitude, and predicted earthquake magnitude, respectively.

Root Mean Square Error (RMSE) is another statistical parameter, which is used in the present study for measuring the accuracy of all models. This parameter shows the differences between the predicted and recorded values, and can be calculated as follows:

$$RMSE = \sqrt{\frac{\sum_{i=1}^{n} (C_o - C_p)^2}{n}}$$
(B2)

Another statistical parameter is Normalized Mean Square Error (NMSE), which is used for assessing the validity of all models in this study. This parameter shows the scatter in the all data set, and can be determined as below:

$$NMSE = \frac{RMSE}{\overline{C_o}}$$
(B3)

Where \overline{C}_0 is the mean of the recorded values. It is clear that the optimal value of the NMSE is zero.

Fraction Bias (FB) is a statistical parameter, which is defined as the normalized mean concentrations. This parameter determines that the model is under-predicting or over-predicting. This parameter can be determined as follows:

$$FB = \frac{(\overline{C}_0 - \overline{C}_p)}{0.5(\overline{C}_0 + \overline{C}_p)}$$
(B4)

Where \overline{C}_{p} is the mean of the forecasted amounts.

Compliance with ethical standards

The authors declare that they have no conflict of interest.

References

- [1] Aven T. On how to define, understand and describe risk. Reliab Eng Syst Saf 2010;95:623–31. doi:10.1016/j.ress.2010.01.011.
- [2] Florido E, Martínez-Álvarez F, Morales-Esteban A, Reyes J, Aznarte-Mellado JL. Detecting precursory patterns to enhance earthquake prediction in Chile. Comput Geosci 2015;76:112–20. doi:10.1016/j.cageo.2014.12.002.
- [3] Špičák A, Vaněk J. Earthquake swarms reveal submarine magma unrest induced by distant megaearthquakes: Andaman Sea region. J Asian Earth Sci 2016;116:155–63. doi:10.1016/j.jseaes.2015.11.017.
- [4] Verdugo R, González J. Liquefaction-induced ground damages during the 2010 Chile earthquake. Soil Dyn Earthq Eng 2015;79:280–95. doi:10.1016/j.soildyn.2015.04.016.
- [5] Keefer DK. Landslides caused by earthquakes. Geol Soc Am Bull 1984;95:406–21.
- [6] Cecioni C, Bellotti G, Romano A, Abdolali A, Sammarco P, Franco L. Tsunami Early Warning System based on Real-time Measurements of Hydro-acoustic Waves. Procedia Eng 2014;70:311– 20. doi:10.1016/j.proeng.2014.02.035.
- [7] Fazendeiro Sá L, Morales-Esteban A, Durand Neyra P. A Seismic Risk Simulator for Iberia. Bull Seismol Soc Am 2016;106:1198–209. doi:10.1785/0120150195.
- [8] Tsai C-W, Lai C-F, Chao H-C, Vasilakos A V. Big data analytics: a survey. J Big Data 2015;2:21. doi:10.1186/s40537-015-0030-3.
- [9] Rouet-Leduc B, Hulbert C, Lubbers N, Barros K, Humphreys CJ, Johnson PA. Machine Learning Predicts Laboratory Earthquakes. Geophys Res Lett 2017;44:9276–82. doi:10.1002/2017GL074677.
- [10] Asencio-Cortés G, Morales-Esteban A, Shang X, Martínez-Álvarez F. Earthquake prediction in California using regression algorithms and cloud-based big data infrastructure. Comput Geosci 2018;115:198-210. doi:10.1016/j.cageo.2017.10.011.
- [11] Wang Q, Jackson DD, Kagan YY. California Earthquakes, 1800-2007: A Unified Catalog with Moment Magnitudes, Uncertainties, and Focal Mechanisms. Seismol Res Lett 2009;80:446–57. doi:10.1785/gssrl.80.3.446.
- [12] Naderpour H, Rafiean AH, Fakharian P. Compressive strength prediction of environmentally friendly concrete using artificial neural networks. J Build Eng 2018;16:213–9. doi:10.1016/j.jobe.2018.01.007.
- Shishegaran A, Ghasemi MR, Varaee H. Performance of a novel bent-up bars system not interacting with concrete. Front Struct Civ Eng 2019;13:1301–15. doi:10.1007/s11709-019-0552-4.
- [14] Shishegaran A, Khalili MR, Karami B, Rabczuk T, Shishegaran A. Computational predictions for estimating the maximum deflection of reinforced concrete panels subjected to the blast load. Int J Impact Eng 2020;139:103527. doi:10.1016/j.ijimpeng.2020.103527.
- [15] Shishegaran A, Daneshpajoh F, Taghavizade H, Mirvalad S. Developing conductive concrete containing wire rope and steel powder wastes for route deicing. Constr Build Mater 2020;232:117184. doi:10.1016/j.conbuildmat.2019.117184.
- [16] Mohammadkhani MR, Shishegaran A, Shokrollahi B. Forecasting probable maximum precipitation using innovative algorithm to estimate atmosphere precipitable water vapor. Math Model Eng 2019;5:90–6. doi:10.21595/mme.2019.20935.
- [17] Shishegaran A, Amiri A, Jafari MA. Seismic performance of box-plate, box-plate with UNP, box-

plate with L-plate and ordinary rigid beam-to-column moment connections. J Vibroengineering 2018;20:1470-87. doi:10.21595/jve.2017.18716.

- [18] Shishegaran A, Rahimi S, Darabi H. Introducing box-plate beam-to-column moment connections. Vibroengineering PROCEDIA 2017;11:200–4. doi:10.21595/vp.2017.18548.
- [19] Reza Ghasemi M, Shishegaran A. Role of slanted reinforcement on bending capacity SS beams. Vibroengineering PROCEDIA 2017;11:195–9. doi:10.21595/vp.2017.18544.
- [20] Naderpour H, Fakharian P. A synthesis of peak picking method and wavelet packet transform for structural modal identification. KSCE J Civ Eng 2016;20:2859–67. doi:10.1007/s12205-016-0523-4.
- [21] Naderpour H, Nagai K, Fakharian P, Haji M. Innovative models for prediction of compressive strength of FRP-confined circular reinforced concrete columns using soft computing methods. Compos Struct 2019;215:69–84. doi:10.1016/j.compstruct.2019.02.048.
- [22] Naderpour H, Fakharian P. Predicting the torsional strength of reinforced concrete beams strengthened with FRP sheets in terms of artificial neural networks. J Struct Constr Eng 2018;5:20–35. doi:10.22065/JSCE.2017.70668.1023.
- [23] Box GEP, Jenkins GM, Reinsel GC, Ljung GM. Time series analysis: forecasting and control. John Wiley & Sons; 2015.
- [24] Matei M. Assessing volatility forecasting models: why GARCH models take the lead. Rom J Econ Forecast 2009;12:42–65.
- [25] Bollerslev T. Generalized autoregressive conditional heteroskedasticity. J Econom 1986;31:307– 27.
- [26] Bates JM, Granger CWJ. The Combination of Forecasts. J Oper Res Soc 1969;20:451–68. doi:10.1057/jors.1969.103.
- [27] Chang JC, Hanna SR. Air quality model performance evaluation. Meteorol Atmos Phys 2004;87. doi:10.1007/s00703-003-0070-7.