A Neuro-Fuzzy Model for Punching Shear Prediction of Slab-Column Connections Reinforced with FRP

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1. Introduction

One of the possible failure modes in reinforced concrete slabs is punching shear [1], and in order to improve the shear behavior of these types of elements, the use of fiber reinforced polymer (FRP) in concrete slabs is expanding. FRP is a material that can use as reinforcement materials or
for external strengthening of existing slabs [2]. They are brittle with a higher strength than steel bars and can improve the punching shear capacities of concrete slabs [3]. Theodorakopoulos and Swamy [4] investigated the effect of FRP on punching behavior in concrete slab-column connections and presented a method to design this type of elements. Zhang and Zhu [5] presented a verified method to finite element analysis of FRP concrete slabs, and for this purpose, they suggested a new element with considering nonlinearity in material and geometric properties. Polak and Lawler [6] studied the retrofit methods of concrete slab-column joints and proposed a procedure based FRP which was improved the ability of these connections in a deal with punching shear. Some researchers investigated the punching shear of slab-column joints reinforced with FRP based on experimental studies [3,7–10] and in some cases, they also presented models to estimate the punching capacity [7,10] or to control it [9]. Nguyen and Rovnak [7] proposed a fracture mechanics based on the obtained results of experimental tests of slabs reinforced with glass FRP to determine the punching shear capacity. They considered several parameters such as depth and size effect to specify the capacity. Pre-stressed FRP plates are considered as a suitable element in strengthening slab-column connections and the ability and positive effects of them on the punching shear capacity has been shown by researchers [8]. The effects of flexural reinforcement and space of shear reinforcement on experimental column-slab connections reinforced with GFRP and steel bars studied by ElGendy [9]. He was found that well-anchored shear studs can control the shear cracks and also the shear failure mode in these slabs. The influence of other parameters such as compressive strength of concrete on FRP-RC slab-column connection was also investigated by the researcher [10], and it was concluded that with increasing this parameter, the capacity of the slab is increased. A state of the art review regarding the punching shear capacity in slab-column joints strengthening with FRP materials has been presented by Saleh et al. [11].

The conventional methods used to determine the performance and behavior of the elements are very time-consuming especially in large and complex problems [12]. For this reason, in recent years, alternative methods such as soft computing approaches (such as neural networks, Fuzzy systems) has been widely used especially in civil engineering [13–26]. They are powerful tools, especially for capacity prediction goals. The application of artificial neural networks to determine the punching shear in concrete slabs reinforced with FRP bars has been investigated by researchers [25,27]. Despite the precision of these models, due to the complexity of the structure of the neural networks, the direct use of them for predictive purposes is very complex and time-consuming. For this reason, other soft computing methods, such as neuro-fuzzy models, are more useful. Such models have the benefits of neural network and fuzzy models simultaneity and are widely used in engineering problems because of their high accuracy and the possibility to provide a mathematical structure based on their solution. In this paper, the authors considered one of the powerful neuro-fuzzy methods namely adaptive neuro-fuzzy inference system (ANFIS) to estimate the punching shear of the slab-column connections reinforced with FRP bars. The presented model is determined based on the experimental tests and the results with high accuracy indicated that the ANFIS model could be used for estimating punching shear. To
increase the workability of the proposed model of this paper, the authors presented their model based on mathematical forms, and for this purpose, the equations and other requirements are also provided in details.

2. Database

Neuro-fuzzy systems are soft computing methods that need a database as an initial knowledge about the system behavior. These types of models are trained based on a series of data from the previous solutions of the problem and be able to estimate the output for an arbitrary input vector. In this paper, the author used a collection of 82 experimental datasets which are published in the literature [7,27,28]. This database was divided into two groups including 64 and 18 datasets for training and testing the neuro-fuzzy model, respectively. Details of the considered datasets are shown in Table 1.

Table 1.
A summary on the considered database.

<table>
<thead>
<tr>
<th>Statistic</th>
<th>Section area of column (mm²)</th>
<th>The effective flexural depth of slab (mm)</th>
<th>Concrete compressive strength (MPa)</th>
<th>Young’s modulus of the FRP slab (GPa)</th>
<th>FRP reinforcement ratio in percent</th>
<th>Punching capacity (KN)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Train Data</td>
<td>Minimum</td>
<td>50.27</td>
<td>55.00</td>
<td>26.00</td>
<td>28.40</td>
<td>0.18</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>2025.00</td>
<td>284.00</td>
<td>118.00</td>
<td>147.60</td>
<td>3.78</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>506.25</td>
<td>131.00</td>
<td>38.60</td>
<td>48.20</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>762.95</td>
<td>138.02</td>
<td>42.13</td>
<td>70.63</td>
<td>0.88</td>
</tr>
<tr>
<td></td>
<td>St.Dev</td>
<td>575.74</td>
<td>58.95</td>
<td>14.00</td>
<td>35.44</td>
<td>0.66</td>
</tr>
<tr>
<td></td>
<td>Mode</td>
<td>400.00</td>
<td>142.00</td>
<td>36.30</td>
<td>48.20</td>
<td>0.73</td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td>1974.73</td>
<td>229.00</td>
<td>92.00</td>
<td>119.20</td>
<td>3.60</td>
</tr>
<tr>
<td>Test Data</td>
<td>Minimum</td>
<td>50.27</td>
<td>61.00</td>
<td>29.00</td>
<td>40.70</td>
<td>0.28</td>
</tr>
<tr>
<td></td>
<td>Maximum</td>
<td>2025.00</td>
<td>284.00</td>
<td>75.80</td>
<td>113.00</td>
<td>3.78</td>
</tr>
<tr>
<td></td>
<td>Median</td>
<td>762.50</td>
<td>131.00</td>
<td>38.60</td>
<td>48.15</td>
<td>1.12</td>
</tr>
<tr>
<td></td>
<td>Average</td>
<td>792.12</td>
<td>146.44</td>
<td>40.41</td>
<td>55.85</td>
<td>1.16</td>
</tr>
<tr>
<td></td>
<td>St.Dev</td>
<td>577.37</td>
<td>66.14</td>
<td>9.99</td>
<td>19.55</td>
<td>0.79</td>
</tr>
<tr>
<td></td>
<td>Mode</td>
<td>900.00</td>
<td>131.00</td>
<td>38.60</td>
<td>42.00</td>
<td>1.21</td>
</tr>
<tr>
<td></td>
<td>Range</td>
<td>1974.73</td>
<td>223.00</td>
<td>46.80</td>
<td>72.30</td>
<td>3.50</td>
</tr>
</tbody>
</table>

The authors considered five parameters as independent variables to determine the punching shear in concrete slab-column connections reinforced with FRP (Fig.1). These parameters are area section of the column, Young’s modulus of the FRP bars, the effective flexural depth of the slab, FRP reinforcement ratio and also the compressive strength of concrete. The considered variables (inputs and output) are presented in Table 2. The normal values are calculated based on the minimum and the range of the parameters, which can be seen in Table 1, to limit the intervals between 0.1 to 0.9. The use of normal value instead of real value can improve the performance of the final ANFIS model.
Table 2.
Definition of the considered parameters.

<table>
<thead>
<tr>
<th>Notation</th>
<th>Description</th>
<th>Normal Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>Section area of column (mm$^2$)</td>
<td>$x_1 = 0.8 \frac{X_1}{50.27} + 0.1$</td>
</tr>
<tr>
<td>$X_2$</td>
<td>The effective flexural depth of slab (mm)</td>
<td>$x_2 = 0.8 \frac{X_2}{229} + 0.1$</td>
</tr>
<tr>
<td>$X_3$</td>
<td>Concrete compressive strength (MPa)</td>
<td>$x_3 = 0.8 \frac{X_3}{26} + 0.1$</td>
</tr>
<tr>
<td>$X_4$</td>
<td>Young’s modulus of the FRP (GPa)</td>
<td>$x_4 = 0.8 \frac{X_4}{28.4} + 0.1$</td>
</tr>
<tr>
<td>$X_5$</td>
<td>FRP reinforcement ratio in percent</td>
<td>$x_5 = 0.8 \frac{X_5}{0.18} + 0.1$</td>
</tr>
<tr>
<td>$V$</td>
<td>Punching capacity of the FRP-reinforced concrete flat slab (kN)</td>
<td>$y = 0.8 \frac{V}{61} + 0.1$</td>
</tr>
</tbody>
</table>

Fig. 1. Distribution of the parameters versus the punching shear.

3. Proposed ANFIS Model

In this paper, ANFIS as a neuro-fuzzy inference system, which was introduced by Jang [29], is considered to determine the punching shear capacities in concrete slab-column connections
reinforced with FRP bars. Each input variables in the proposed ANFIS structure have four Gaussian membership functions (MF). The selected algorithm to generate ANFIS here was subtractive clustering. In this method, datasets are assigned to groups called clusters to determine the existing patterns in the system. To estimate the unknown parameters of the ANFIS (membership function parameters and also the rules and the coefficients of the linear output functions of each rule), the model was trained based on 64 datasets and then was evaluated by 18 separated datasets as the testing data. Details of the ANFIS is presented and formulated in the next section. The membership functions of the inputs are illustrated in Fig. 2. The final structure of the ANFIS with four rules is also shown in Fig. 3.

Fig. 2. Membership functions of the input parameters.
As previously mentioned, the presented neuro-fuzzy model of the ANFIS in this paper used four Gaussian membership functions (MF) for each of its input parameters. A Gaussian function can be shown by Eq. 1 (x is the input, c is the mean and σ is the variance for x). Details of the membership functions are presented in Table 3. Also, there were four linear functions (Eq. 2) for the proposed ANFIS with the parameters of Table 4.

\[ MF_i(x: s, c) = e^{-\frac{(x - c)^2}{2\sigma^2}} \]  \hspace{1cm} (1)

\[ C_j = a_1x_1 + a_2x_2 + a_3x_3 + a_4x_4 + a_5x_5 + a_0 \]  \hspace{1cm} (2)

| Table 3. |
|---|---|---|---|---|---|
| Input Functions | MF1 | Input Functions | MF2 | Input Functions | MF3 | Input Functions | MF4 |
| \( \sigma \) | 0.1530 | \( \sigma \) | 0.1319 | \( \sigma \) | 0.1038 | \( \sigma \) | 0.1103 |
| \( c \) | 0.6754 | \( c \) | 0.6177 | \( c \) | 0.2824 | \( c \) | 0.1965 |
| \( \sigma \) | 0.0906 | \( \sigma \) | 0.1105 | \( \sigma \) | 0.0897 | \( \sigma \) | 0.1029 |
| \( c \) | 0.4774 | \( c \) | 0.5123 | \( c \) | 0.3234 | \( c \) | 0.2527 |
| \( \sigma \) | 0.0575 | \( \sigma \) | 0.0682 | \( \sigma \) | 0.0476 | \( \sigma \) | 0.0455 |
| \( c \) | 0.2843 | \( c \) | 0.3146 | \( c \) | 0.1927 | \( c \) | 0.2109 |
| \( \sigma \) | 0.0842 | \( \sigma \) | 0.1737 | \( \sigma \) | 0.0944 | \( \sigma \) | 0.1433 |
| \( c \) | 0.2171 | \( c \) | 0.7239 | \( c \) | 0.2159 | \( c \) | 0.5824 |
| \( \sigma \) | 0.0753 | \( \sigma \) | 0.0475 | \( \sigma \) | 0.0841 | \( \sigma \) | 0.0696 |
| \( c \) | 0.3060 | \( c \) | 0.1936 | \( c \) | 0.2849 | \( c \) | 0.1987 |

| Table 4. |
|---|---|---|---|---|---|
| Input Functions | C1 | Input Functions | C2 | Input Functions | C3 | Input Functions | C4 |
| \( a_1 \) | 0.2323 | \( a_2 \) | 1.1320 | \( a_3 \) | 0.3688 | \( a_4 \) | -0.3967 | \( a_5 \) | 0.0072 | \( a_0 \) | -0.2640 |
| \( a_1 \) | -0.1913 | \( a_2 \) | 1.0020 | \( a_3 \) | 0.4249 | \( a_4 \) | 0.7834 | \( a_5 \) | 1.1320 | \( a_0 \) | -0.7735 |
| \( a_1 \) | 0.3199 | \( a_2 \) | 0.1540 | \( a_3 \) | 0.0009 | \( a_4 \) | 0.2108 | \( a_5 \) | 0.0798 | \( a_0 \) | -0.0188 |
| \( a_1 \) | -0.0184 | \( a_2 \) | 0.7607 | \( a_3 \) | 0.5423 | \( a_4 \) | -0.0377 | \( a_5 \) | 0.2791 | \( a_0 \) | -0.1369 |
4. Mathematical Steps to Predict the Punching Shear

In this section, the proposed ANFIS model is expressed in mathematical steps to provide a more efficient and simple operation of the model. The steps are as follow:

Step 1. Determine the normal value of each input variable based on the equations of Table 2.

Step 2. Determine the weight of each four rules based on Eqs. 3:

\[
W_1 = \begin{pmatrix}
0.0468 & 0.0164 & 0.0066 & 0.0142 & 0.0113 \\
0.6754 & 0.4774 & 0.2843 & 0.2171 & 0.3060
\end{pmatrix}
\]

\[
W_2 = \begin{pmatrix}
0.0348 & 0.0244 & 0.0093 & 0.0063 & 0.0045 \\
0.6177 & 0.5123 & 0.3146 & 0.7239 & 0.1936
\end{pmatrix}
\]

\[
W_3 = \begin{pmatrix}
0.0215 & 0.0161 & 0.0045 & 0.0178 & 0.0141 \\
0.2824 & 0.3234 & 0.1927 & 0.2159 & 0.2849
\end{pmatrix}
\]

\[
W_4 = \begin{pmatrix}
0.0243 & 0.0212 & 0.0041 & 0.0411 & 0.0097 \\
0.1965 & 0.2527 & 0.2109 & 0.5824 & 0.1987
\end{pmatrix}
\]

Step 3. Determine the output linear functions of each rule by Eqs. 4:

\[
\begin{align*}
C_1 &= 0.2323x_1 + 1.1320x_2 + 0.3688x_3 + 0.3967x_4 + 0.0072x_5 + 0.2640 \\
C_2 &= 0.1913x_1 + 1.0020x_2 + 0.4249x_3 + 0.7834x_4 + 1.1320x_5 + 0.7735 \\
C_3 &= 0.3199x_1 + 0.1540x_2 + 0.0009x_3 + 0.2108x_4 + 0.0798x_5 + 0.0188 \\
C_4 &= 0.0184x_1 + 0.7607x_2 + 0.5423x_3 + 0.0377x_4 + 0.2791x_5 + 0.1369
\end{align*}
\]

Step 4. Determine the output of the ANFIS by Eq. 5. It is worth noting that the output of the ANFIS is based on the normal value of the target, which is between 0.1 to 0.9. Therefore, it needs to convert to its corresponding real value by Eq. 6.

\[
\begin{align*}
0.1 & \leq \left( Y = \frac{W_1C_1 + W_2C_2 + W_3C_3 + W_4C_4}{W_1 + W_2 + W_3 + W_4} \right) \leq 0.9 \\
V(kN) &= \frac{1539Y - 153.9}{0.8} + 61
\end{align*}
\]
5. Results of the Proposed Model

In this section, the obtained results of the proposed ANFIS are presented. The considered laboratory datasets contain 82 datasets, which were divided into two groups of 64 and 18 datasets for training and testing phases, respectively. The reason for using test data was evaluating the final proposed ANFIS model. The results of the training phase for all 82 datasets is presented in Fig. 4. It is clear from the figure that the model with suitable accuracy trained well. The amount of root mean square error (RMSE) and also error mean were 0.025404 and -1.86e-08, which is indicated that the ANFIS determined the output of 82 datasets with low error. This issue is illustrated in the figure for each of dataset.

![Fig. 4. Results of the training datasets.](image)

After training the ANFIS, it should test by several new datasets. For this purpose, the authors used 18 datasets. The results of the proposed ANFIS structure for the testing phase is shown in Fig. 5 and it is indicated that the model can estimate the punching shear capacities of the test data with high accuracy. RMSE and error mean for these datasets were 0.04345 and 0.016371, respectively. For better comparison, the regression plots of the training and testing phases of the proposed ANFIS model is also presented in Fig. 6. It can be seen from the figure that the correlation coefficients for both phases were more than 0.98.
6. Conclusion

Determination of capacity of structural elements in civil engineering is an open issue that has widely considered in research and studies. In recent years, soft computing methods are used for predicting goals instead of traditional approaches such as regressions. They are simple, flexible and powerful and their abilities are proved in several studies. In this paper, one of this method, which is called ANFIS is considered to estimate the capacity of concrete column-slab connections reinforced with FRP. A collection of 82 experimental datasets that are published in literature was used to train and also test the final structure of the ANFIS. The results of the proposed model showed that ANFIS could be able to present the target with high accuracy. The
The presented model considered five inputs as the independent variables and used four Gaussian membership function in its structure. For simplicity, the authors also presented their model in a mathematical form by some equations. The output results of the equations are compared with the experimental results, and it was concluded that the predicted values are very close to the experimental values. The amounts of RMSE, R2 and also error mean for both phases of training and testing confirm the performance of the proposed ANFIS for the considered goal.

References