Prediction of Ultimate Bearing Capacity of Skirted Footing Resting on Sand Using Artificial Neural Networks

R. Kumar Dutta¹, R. Rani¹ and T. Gnananandarao¹*
1. Department of Civil Engineering, NIT Hamirpur, Himachal Pradesh, India

Corresponding author: anandrcwing@gmail.com
http://dx.doi.org/10.22115/SCCE.2018.133742.1066

1. Introduction

For the reasonable and economical design of the footing, correct estimation of the ultimate bearing capacity was essential in geotechnical engineering. Many experimental investigations on different regular shaped skirted footings resting on sand have been reported in literature. Further, for these types of footings, no bearing capacity equation is available in literature and the researchers have to resort to costly experimental investigations. In this context, there is need to
have a mathematical model where one can calibrate and fit the experimental data in order to make out the relationships among the participating parameters. Artificial neural network (ANN) without any prior assumption has the capability to store, learn and capture the intricate relationship among the participating parameters. This capability of neural network makes it a better choice for modelling the ultimate bearing capacity of different regular shaped skirted footings resting on sand. A neural network model to predict the ultimate bearing capacity of different regular shaped skirted footings resting on sand from the data collected from literature has been presented in this paper. The input parameters used were normalised skirt depth, area of the footing and the friction angle of the sand and the output was the ultimate bearing capacity. Sensitivity analysis has been performed on the parameters affecting the ultimate bearing capacity. Comparison of the ANN model was attempted with the model obtained from the multiple regression analysis (MRA). Model equation based on the connection weight obtained in the neural network analysis was finally proposed in this paper.

2. Background

Many experimental studies on the different regular shaped skirted footing resting on sand have been reported in literature [1-13]. In the recent past, several studies [14-30] in literature using ANN have been reported. Researchers predicted the ultimate bearing capacity of the regular shaped such as circular [31], strip [32, 33], spread [34] footing resting on sand/clay/rock using ANN. These studies have concluded that the ANN model was a dependable tool for the prediction of the bearing capacity in all the soil conditions and for all footing shapes. Researcher’s [1, 4-5] reported that the provision of the skirts to the regular shaped footings were performed better in comparison to the conventional footings in terms of bearing capacity. These studies have revealed that the skirts attached to the regular shaped footings have improved the bearing capacity and reduced in the settlement. The bearing capacity of thin walled spread footing using different soft computing technique such as artificial neural network [35], particle swarm optimization [36] and adaptive neuro-fuzzy inference system [37] are available in literature. These studies have proven the efficacy of soft computing techniques in prediction of the bearing capacity of the thin-walled spread footing. But no study has been reported to predict the ultimate bearing capacity of different regular shaped footing with skirt and resting on sand in literature. This paper tries to fill this gap. The paper presents an ANN model to predict the ultimate bearing capacity of different regular shaped footings with skirt and resting on sand from the data collected from literature. The input parameters considered were normalised skirt depth, area of the footing and the friction angle of the sand and ultimate bearing capacity was the output.

3. Artificial Neural Network

The methodology used for obtaining the model for prediction of the ultimate bearing capacity of the different regular shaped skirted footings resting on sand using artificial neural networks has been presented in this section. The process began with the selection of the input and the output parameters. The input parameter considered in this investigation were normalised skirt depth
(D/B), area of the footing (A), friction angle (φ) of the sand and ultimate bearing capacity (q_ult) was the output. It is pertinent to mention here that the performance of any of the ANN model depends upon the network architecture as well as on the parameter settings as reported by [38]. Researchers left with no option except to adopt a trial and error procedure to get the structure of the network. Assignment of initial weights as well as other related parameters also affects the efficiency of the neural network. Choosing the number of hidden layers and the number of neuron in each layer was a complex phenomenon for a multilayer feed-forward network. Number of nodes in the hidden layer shall be decided first before the selection of the number of the hidden layers. It was reported by [38] that the number of neuron in the hidden layer shall be 2/3 or 70% of the size of the input layer. Further, if the number of neuron in the hidden layer was inadequate then number of neuron in the output layer can be added later. It was reported by [39] that the number of neuron in the hidden layer shall be less than twice of the number of neuron present in the input layer. The size of the neuron in the hidden layer shall be kept between the input and the output layer size as reported by [40]. In view of the above, the number of neuron in the hidden layer was calculated using the rule proposed by [38] in this paper. The topology of the ANN model representing 3-2-1 architecture was shown in Fig. 1. The data used for training and testing of the ANN model has been collected from [1, 2, 4, 12, 41] and the range of the input and output parameters were presented in Table 1. To develop the ANN model, the data was selected for the training and the testing randomly. For training purposes, 70% of the total data (149) was taken and the remaining 30% was utilized for testing the model. Similar percentage of the data for the training and the testing purpose were used by [28].

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Minimum</th>
<th>Maximum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normalised skirt depth (D/B)</td>
<td>0</td>
<td>2</td>
</tr>
<tr>
<td>Area of the footing (A, m²)</td>
<td>0.00126</td>
<td>0.00785</td>
</tr>
<tr>
<td>Friction angle of the sand (φ, Deg.)</td>
<td>27</td>
<td>44.60</td>
</tr>
<tr>
<td>Ultimate bearing capacity (q_ult, kPa)</td>
<td>42</td>
<td>8005</td>
</tr>
</tbody>
</table>

In order to select the optimum no of iterations, trials were performed and the plot was made with the number of iterations on the horizontal axis and the mean square error (MSE) on the vertical axis as shown in Fig. 2.
Study of Fig. 2 indicates that the number of iterations corresponding to the minimum MSE was 650. The next step was to choose the learning rate which has been taken as 0.7 (default value in the open source Agiel neural network software). This software contains 18 different activation functions. By adopting a trial and error procedure, choosing the different activation function and measuring the performance measures such as coefficient of correlations, coefficient of determination, mean squared error, relative mean squared error, mean absolute error and mean absolute percentage error of each of the model and comparing these performance measures with each of the activation function gave sigmoid activation as the best activation function among all the 18 activation functions present in the Agiel neural network software to predict the ultimate bearing capacity.

3.1. Sensitivity analysis

In order to study the relative influence of the input parameters on the ultimate bearing capacity, a sensitivity analysis was carried out. The connection weight approach reported by [43] has been used for this purpose and the following governing equation was used.

\[ RI_j = \sum_{k=1}^{h} (w_{jk} \times w_k) \]  

(1)

Where, \( w_{jk} \) is the connection weight between \( j^{th} \) input parameter and \( k^{th} \) neuron of the hidden layer, \( w_k \) is the connection weight between \( k^{th} \) neuron of hidden layer and the single output neuron, \( RI_j \) is the relative importance of the \( j^{th} \) neuron of input layer and \( h \) is the number of neurons in the hidden layer.

The chart showing the relative importance of the individual input variable considered in the sigmoid activation function based 3-2-1 architecture was shown in Fig. 3.
Fig. 3 reveals that the normalised skirt depth (50 %) was the most important parameter affecting the ultimate bearing capacity of the different regular shaped skirted footing followed by area of the footing (30 %) and the friction angle of the sand (20 %).

3.2. Model equation

Development of the model equation to predict the ultimate bearing capacity of the different regular shaped footings with skirt and resting on sand has been presented in this section. From the ANN analysis, it was found that the sigmoid activation function was the best among the 18 activation functions available in the Agiel neural network software. The obtained hidden weights and the biases between the input and the output parameter were shown in Table 2. Basic output function in the ANN model was chosen as given below in equation (2).

\[ q_{ult} = f \left\{ b_0 + \sum_{k=1}^{h} \left( w_k \times f \left[ b_{hk} + \sum_{j=1}^{m} \left( w_{jk} \times X_j \right) \right] \right) \right\} \]  

The equation derived for the ultimate bearing capacity was based on the trained weights and biases as shown in Table 2. The obtained equations were as under:

Table 2 Final weight between the input and the hidden neuron as well as the hidden neuron and the output neuron including biases

<table>
<thead>
<tr>
<th></th>
<th>weights(w)</th>
<th>biases</th>
</tr>
</thead>
<tbody>
<tr>
<td>D/B</td>
<td>A</td>
<td>φ</td>
</tr>
<tr>
<td>-3.80</td>
<td>0.29</td>
<td>-4.60</td>
</tr>
<tr>
<td>-3.26</td>
<td>-5.66</td>
<td>3.13</td>
</tr>
</tbody>
</table>

\[ X = 6.56 - 3.8 \times \frac{D}{B} + 0.29 \times A - 4.60 \times \phi \]  

(3)
\[ Y = 3.86 - 3.26 \times \frac{D}{B} - 5.66 \times A + 3.13 \times \phi \]  
\hspace{4cm}(4)  

\[ Z = 4.09 - \left( \frac{5.71}{1+\left(1+e^z\right)^{-1}} \right) - \left( \frac{3.91}{1+\left(1+e^z\right)^{-1}} \right) \]  
\hspace{4cm}(5)  

\[ q_{ult} = \frac{1}{1+\left(1+e^{-z}\right)} \]  
\hspace{4cm}(6)  

The \( q_{ult} \) obtained from the above equation will be in the range of [0 to 1] as the activation function used for the ANN modelling was sigmoid. In order to obtain the actual ultimate bearing capacity, the de-normalization of the above equation (6) has been carried out as given below.

\[ q_{ult\text{ actual}} = 0.5(q_{ult} + 1)(q_{ult\text{ max}} - q_{ult\text{ min}}) + q_{ult\text{ min}} \]  
\hspace{4cm}(7)  

Where \( q_{ult\text{ max}} \) is the maximum predicted ultimate bearing capacity, \( q_{ult\text{ min}} \) is the minimum predicted ultimate bearing capacity respectively. The plots were drawn for the training and the testing data of the ANN model and were shown in the Fig. 4 and Fig. 5 respectively. The coefficient of determination \( (R^2) \) of the Fig. 4 and Fig. 5 reveals that the ANN model was able to predict the ultimate bearing capacity of the different regular shaped skirted footing resting on sand within acceptable accuracy.

![Graph](image.jpg)

**Fig. 4.** Predicted and target ultimate bearing capacity of the different regular shaped skirted footing resting on sand in the training data using sigmoid activation function with 3-2-1 topology.
3.3. Multiple regression analysis

The non-linear MRA has been carried out on the total data set using Datafit 9.0 (trial version) software and the equation obtained from this analysis was shown below as equation (8) and the calculated performance measures were tabulated in Table 3.

\[
q_{ult/r} = e^{\left(1.29 \times \frac{D}{B} + 298.48 \times A + 0.13 \times \phi - 1.56\right)}
\]

(8)

Table 3.
Comparison between performance measures of ANN model and MRA model.

<table>
<thead>
<tr>
<th>Performance measures</th>
<th>ANN Training</th>
<th>ANN Testing</th>
<th>MRA Training</th>
<th>MRA Testing</th>
</tr>
</thead>
<tbody>
<tr>
<td>r</td>
<td>0.90</td>
<td>0.96</td>
<td>0.93</td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.92</td>
<td>0.98</td>
<td>0.80</td>
<td></td>
</tr>
<tr>
<td>MSE</td>
<td>102685.10</td>
<td>20804.46</td>
<td>216750.16</td>
<td></td>
</tr>
<tr>
<td>RMSE</td>
<td>320.45</td>
<td>144.24</td>
<td>465.56</td>
<td></td>
</tr>
<tr>
<td>MAE</td>
<td>195.16</td>
<td>81.61</td>
<td>295.83</td>
<td></td>
</tr>
<tr>
<td>MAPE</td>
<td>42.14</td>
<td>30.45</td>
<td>89.20</td>
<td></td>
</tr>
</tbody>
</table>

Study of Table 3 indicates that the r and R² of the MRA model was less in comparison to one obtained using ANN model which indicate that the ANN model has better prediction accuracy in comparison to the MRA model. The predicted versus targeted ultimate bearing capacity of the different regular shaped skirted footings resting on sand using MRA was shown in Fig. 6. The coefficient of determination (R²) of the Fig. 6 reveals that the proposed model equation using
MRA was also reasonably predicting the ultimate bearing capacity of the different regular shaped skirted footing resting on sand. It is pertinent to mention here that majority of the data collected from the literature for the different regular shaped skirted footing were for the loose sand and very less number of data were for the dense sand or footing with higher normalised skirt depth resulting concentration of more data in one area and lesser concentration of the data in other area as evident from Fig. 6.

![Predicted vs Targeted Ultimate Bearing Capacity](image)

**Fig. 6.** Predicted and target ultimate bearing capacity of the different regular shaped skirted footing resting on sand in the total data using MRA.

### 4. Comparison

In order to assess the accuracy of the proposed ANN model equation developed using trained weights and biases of the sigmoid function, the comparison was made with the ultimate bearing capacity model obtained from the MRA model and the results were tabulated in Table 3 and plotted in Fig. 7.
A close examination of Table 3 reveals that the coefficient of correlation and the coefficient of determination of the data was increased by 3.12 % and 18.37 % whereas the MSE, RMSE, MAE and MAPE was decreased by 90.40 %, 69.02 %, 72.41 % and 65.86 % respectively in comparison to the MRA. The higher coefficient of determination obtained for the ANN model as shown in Fig. 7 reveals its superiority over the MRA model. This is attributed to the fact that the MRA model may not compete well with the non-linear data whereas ANN model was suitable for the non-linear data for prediction.

5. Conclusion

Predicting the ultimate bearing capacity of the different regular shaped footing with skirt and resting on sand was a complex phenomenon. Experimental methods for the determination of the ultimate bearing capacity turned out to be highly expensive in terms of cost and time. In order to overcome this problem an attempt was made using ANN to predict the ultimate bearing capacity of the different regular shaped footing with skirt and resting on sand. Model equation has been developed which gave reasonably acceptable results and was compared with the MRA model. The study bring forward the following conclusion.

1. ANN model with 3-2-1 topology has accurately predicted the ultimate bearing capacity of the different regular shaped footing with skirt and resting on sand.
2. Sigmoid activation function performed the best among the 18 activation and was able to predict the ultimate bearing capacity of the different regular shaped footings with skirt and resting on sand closer to the actual one.
3. Sensitivity analysis revealed that the normalised skirt depth (50 %) followed by area of the footing (30 %) and the friction angle of the sand (20 %) was affecting the ultimate bearing capacity in this order.
4. The coefficient of correlation and coefficient of determination of the data was increased by 3.12 % and 18.37 % whereas the MSE, RMSE, MAE and MAPE was decreased by 90.40 %
%, 69.02 %, 72.41 % and 65.86 % respectively in comparison to the multiple regression analysis.

5. ANN model equation was proposed for the prediction of the ultimate bearing capacity of different regular shaped footing with skirt and resting on sand.

6. The developed MRA model predicts the ultimate bearing capacity of the different regular shaped footing with skirt and resting on sand with ease as the developed equation was quite simple in comparison to the ANN model. However the prediction was likely to be less accurate in comparison to the ANN model.

**Notations**

- $A$: Area of footing
- $\phi$: Friction angle of the sand
- $D/B$: Normalised skirt depth
- $q_{\text{ult}}$: Ultimate bearing capacity
- $\text{MRA}$: Multiple regression analysis
- $\text{ANN}$: Artificial Neural network
- $b_{hk}$: Bias at the $k^{th}$ neuron of the hidden layer
- $b_o$: Bias at the output layer
- $f$: Optimum activation function
- $\text{MAE}$: Mean absolute error
- $\text{MAPE}$: Mean absolute percentage error
- $\text{MSE}$: Mean square error
- $q_{\text{ult}}/r$: Actual ultimate bearing capacity
- $q_{\text{ult}}/\text{max}$: Maximum ultimate bearing capacity
- $q_{\text{ult}}/\text{min}$: Minimum ultimate bearing capacity
- $q_{\text{ult}}/r$: Regression model ultimate bearing capacity
- $r$: Correlation coefficient
- $R^2$: Coefficient of determination
- $RI_j$: Relative importance of the $j^{th}$ neuron of input layer
- $\text{RMSE}$: Root mean square error
- $W_{jk}$: Connection weight between $j^{th}$ input variable and $k^{th}$ neuron of
- $W_k$: Connection weight between $k^{th}$ neuron of hidden layer and the single
- $X_j$: Normalized input variable $j$ in the range $[0, 1]$

**References**


