Cone Penetration Based Probabilistic Assessment of Shallow Foundation Settlement

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ABSTRACT

Probabilistic (Reliability or safety) analysis, as a measure of structural performance, was expressed in terms of reliability indices which were calculated for a total settlement of shallow foundations in a Site in Abuja, the Federal Capital of the Federal Republic of Nigeria based on the Burland and Burbidge settlement prediction method. Reliability indices were calculated with the objective of developing a risk analysis procedure specifically for prediction of the settlement of foundations lying on soils. This research was aimed at the development of a method that will assist in the process of calibration of load and resistance factors (reliability-based design (RBD)) for service limit state based on cone penetration test (CPT) results. The CPT data were obtained from four test holes (CPT1 - 4) at three foundation embedment depths of 0.6, 1.2 and 1.8 m and analysis was done using applied foundation pressures of 50, 100, 200, 300 and 500 kN/m². Reliability analysis, expressed in the form of reliability index (β) and the probability of failure (Pf) was performed for foundation settlement using the First Order Reliability Method (FORM) in MATLAB. The footings were designed for a 25 mm allowable settlement value as recommended in Eurocode 7 for serviceability limit state (SLS) design which is a conventional approach. Sensitivity study indicated that the applied foundation pressure and coefficient of variation (COV) of CPT tip resistance significantly affected the magnitude of foundation settlements and the variability of the geotechnical parameters is highly influenced and has a significant effect on the settlement and safety of any structure. The use of COV value of 30 % of CPT tip resistance which corresponds to target reliability index (βT) of 4.52 and target probability of failure (PfT) of 0.000677% based on the Burland and Burbidge method for SLS design is recommended for RBD of footings total settlement on soils in Abuja, Nigeria.
1. Introduction

Design of structural foundation consists of selecting and proportioning foundations in a manner that limit states are not exceeded. Limit states are of two types: ultimate limit states ULS and serviceability limit states SLS. ULSs are associated with danger, involving such outcomes as structural collapse. SLSs are associated with impaired functionality, and, in foundation design, are often caused by bearing capacity failure and excessive settlement. Reliability-based design (RBD) is a design philosophy that aims at ensuring that the probability of reaching limit states is lower than some limiting value. Thus, a direct assessment of risk is possible with RBD. This evaluation is not achievable using the traditional working stress design methodology [1]. Many code-writing organizations have developed the load factors; ASCE, ACI, and AASHTO [2]. However, a reliable set of resistance factors is required for geotechnical LRFD. Reliability-based design tools can be suitably used to develop these resistance factors. Reliability of the system is the relationship between loads the system must carry and its ability to carry the load. Reliability of the system is expressed in the form of reliability index ($\beta$) which is related to the probability of failure of the system ($P_f$). In this study, a reliability analysis was performed for foundation settlement using the First Order Reliability Method (FORM).

Soil composition and properties vary from one location to another, even within homogeneous layers. The variability is attributed to factors such as variations in mineralogical composition, conditions during deposition, stress history, and physical and mechanical decomposition processes [3]. The spatial variability of soil properties is a major source of uncertainty. Spatial variability is not a random process; rather it is controlled by location in space. Statistical parameters such as the mean and variance are one-point statistical parameters and cannot capture the features of the spatial structure of the soil [4]. In geotechnical engineering, the bearing capacity and settlement of foundation are traditionally evaluated by a deterministic (empirical) approach. The factor of safety used in the deterministic approach accounts for natural variability, statistical uncertainty, measurement errors, and limitations of analytical models and is an indirect way of limiting deformation [5,6]. Thus the factor of safety used in the deterministic approach does not consider the sources and amount of uncertainty associated with the system [7,8].

In geotechnical engineering, the bearing capacity and settlement of foundation were traditionally evaluated by a deterministic (empirical) approach. The factor of safety used in the deterministic approach accounts for natural variability, statistical uncertainty, measurement errors, and limitations of analytical models and is an indirect way of limiting deformation [6]. A factor of safety of 2.5 to 3.0 is generally adopted to account for this variability [9]. Over the last two decades, there has been a slow but worldwide shift toward the increased use of risk-based design methodologies for geotechnical engineering. The increasing awareness that soils are materials
that, even in a lithologic homogeneity, show pronounced variability in their physicomechanical properties, has caused a remarkable increase to the efforts to develop probabilistic computational models in geotechnical engineering [10]. Needs for carrying out reliability analysis (RA) for complex geotechnical design problems are increasing due to the introduction of the limit state design worldwide. On the other hand, in the current practical design of geotechnical structures, many sophisticated calculation methods, e.g., commercially available user-friendly FEM programs etc., are employed [11]. A rational quantification and incorporation of uncertainty into the design process is allowed in probabilistic analyses.

Cone Penetrometer Testing (CPT) provides a rapid and economical means by which the objective of all site investigations, which is to obtain data that can adequately quantify the variability of the geotechnical properties of the site can be achieved. Details on the procedure of CPT can be found in ASTM D-3441 [12]. The study was aimed at the development of a methodology to assist in the process of calibration of load and resistance factors for service limit state.

In the literature, the selected target probability of failure (P_T) for the serviceability limit state (SLS) design of footings varies considerably. To evaluate the factors of deformation for settlement design of footings on the sand, Fenton et al. [13] used a maximum target probability of failure (P_T) of 5 %, which corresponds to a reliability index of 1.645. Popescu et al. [14] also used 5 % as the P_T for both differential settlement and bearing capacity. A P_T value as high as 30 % was reported by Zekkos et al. [15]. For all of the studies reported, the probability of failure is high. In a study on the reliability analysis of settlement for shallow foundations in bridges by Ahmed [16], target reliability index (β_T) of 3.5 which corresponds to probability of exceeding the limit (Targeted probability of failure, P_T) of 0.02 % for total settlement was recommended related with allowable suggested total settlement value of 37.5 mm. For allowable settlement of 40 mm, Subramaniam [17] reported a reliability index of 2.83 corresponding to the probability of failure of 0.23% and based on the allowable settlement of 25 mm, Salahudeen et al. [18–20] reported a target reliability index of 3.15 corresponding to the probability of failure of 0.0789%.

Taking foundation movement analyses into account for tower structures together with structure-foundation interaction and precedents, a target reliability index of 2.6, was recommended by Phoon et al. [5] which corresponds to a P_T of about 0.47% for the SLS design of footings. However, considering the subjectivity inherent in SLS design, this target probability of failure (P_T) can only be considered as an estimate [5]. This P_T value could be reduced for less restrictive design conditions or where uncertainty is reduced significantly due to circumstances such as local experience with the soil conditions. Conversely, it could be increased for more restrictive design conditions with a high level of uncertainty [21]. The selected value of P_T should be consistent with the implied safety levels in the existing designs.
2. Methodology

The study made use of cone penetration test (CPT) data collected from four test holes in Africa Development Bank Field Office Site, Abuja, the Federal Capital of the Federal Republic of Nigeria. Foundation settlement estimates were made at depths of 0.6, 1.2 and 1.8 m and applied foundation pressures of 50, 100, 200, 300 and 500 kN/m². The reliability analysis was performed using the First Order Reliability Method (FORM) in MATLAB [22] Programme FORM that uses the first terms of a Taylor series expansion to estimate the mean value and variance of performance function is called First Order Second Moment (FOSM) reliability method because the variance is in the form of the second moment. The methodology of the FOSM reliability method in detail is described in Baecher and Christian [23].

Optimization was performed with the aid of genetic algorithm which drives biological evolution. The genetic algorithm repeatedly modifies a population of individual typically random chromosomes. This study made use of 1000 runs (number of genetic algorithms).

The limit state function is defined as a function of capacity and demand; it is denoted as g and expressed as:

\[ g(R, Q) = R - Q \]  

Where R, is the structural resistance or capacity of the structural component and Q is the load effect or demand of the structural component with the same units as the resistance. The performance function g(X) is a function of capacity and demand variables \( X_1, X_2, \ldots, X_n \) which are basic random variables for both R and Q such that:

\[
g(X_1, X_2, \ldots, X_n) \begin{cases} > 0 & \text{safe state} \\ = 0 & \text{limit state} \\ < 0 & \text{failure state} \end{cases}
\]  

where \( g(x) = 0 \) is known as a limit state surface, and each X indicates the basic load or resistance variable.

The probability of failure, \( P_f \), can be related to an indicator called the reliability index, \( \beta \). For the estimation of the probability of failure, the method employed involves approximate iterative calculation procedures. Using this method, two useful procedures were considered [24]:

\( a) \) Expectations: \( \mu_i = E[X_i], i = 1, \ldots, n \)

\( b) \) Covariances: \( C_{ij} = Cov[X_i, X_j], i, j = 1, 2, \ldots, n \)
The “safety margin” is the random variable $M = g(x)$ (also called the ‘state function’). Non-normal variables are transformed into independent standard normal variables, by locating the most likely failure point, $\beta$-index (called the reliability index), through an optimization procedure. This is also done by linearizing the limit state function in that point and by estimating the failure probability using the standard normal integral.

The reliability index, $\beta$, is then defined by Hasofer and Lind [25] as:

$$
\beta = \frac{\mu_m}{\sigma_m}
$$

(5)

where $\mu_m$ = mean of $M$

and $\sigma_m$ = Standard deviation of $M$

If $R$ and $S$ are uncorrelated and with $M = R - S$, then

$$
\mu_m = \mu_R - \mu_S \quad \text{and} \quad \sigma_m^2 = \sigma_R^2 + \sigma_S^2
$$

(6)

Therefore,

$$
\beta = \frac{\mu_R - \mu_S}{(\sigma_R^2 + \sigma_S^2)^{1/2}}
$$

(7)

2.1. The performance function

A relationship was established between the probability of failure, $P_f$, and the safety index, $\beta$. This relationship holds only when the safety margin, $M$, is linear in the basic variables, and these variables are normally distributed. This relationship is stated below:

$$
P_f = -\Phi(-\beta)
$$

(8)

and

$$
\beta = -\Phi^{-1}(P_f)
$$

(9)

where $\Phi$ is the standardized normal distribution function.

$$
P_f = P\{(R - S) \leq 0\} = P(M \leq 0) = \Phi\left(\frac{0-(\mu_R-\mu_S)}{\sqrt{\sigma_R^2 + \sigma_S^2}}\right) = \Phi(-\beta)
$$

(10)

The performance function used for this study is:
\[ G(X) = S_e - \left[ (0.14 \times \alpha \times B_R) \left( \frac{1.71}{N^{0.4}} \right) \left( \frac{1.25 \left( \frac{L}{B} \right)}{0.25 + \left( \frac{L}{B_R} \right)} \right)^2 \left( \frac{H}{B_R} \right)^{0.7} \left( \frac{q}{P_a} \right) \right] \]  

where:

\[ N = \frac{q_c}{7.6429 \times P_a \times D_{50}^{0.26}} \]

\[ G(X) = \text{Performance function} \]
\[ S_e = \text{Allowable settlement} = 25 \text{ mm} \]
\[ N_{60(a)} \approx 15 + 0.5(N_{60} - 15) \]
\[ N_{60(a)} = \text{Adjusted} \ N_{60} \ \text{value} \]
\[ B_R = \text{Reference width} = 0.3 \text{ m} \]
\[ B = \text{Width of the actual foundation} (\text{m}) \]
\[ \alpha = \text{Depth of stress influence correction factor} \]
\[ H = \text{Thickness of the compressible layer} (\text{m}) \]
\[ L = \text{Length of foundation} (\text{m}) \]
\[ q = \text{Applied foundation pressure} (\text{kN/m}^2) \]
\[ P_a = \text{Atmospheric pressure} = 100 \text{ kN/m}^2 \]

After the performance function \(G(x)\) and the underlying random variables have been defined, the probability of failure \((P_f)\) and the reliability index \((\beta)\) were evaluated for each design case using the methodology described herein. In this study, the footings were designed for a 25 mm allowable settlement value as recommended in Eurocode 7 [26] for serviceability limit state (SLS) design of footings which is the average value that can be encountered in practice. If the limiting value (25 mm) is exceeded, it is likely to cause the occurrence of an ultimate limit state (ULS). In the Burland and Burbidge [27] method used in this study, \(\alpha, P_a, B,\) and \(L\) are assumed to be deterministic values. The random variables considered are the SPT \(N_{60}\) (derived from CPT cone resistance) value and the applied foundation pressure. The flow chart for the reliability analysis procedure used in this study is shown in Figure 1.
3. Results and discussion

3.1. Foundation settlement and CPT tip resistance

For shallow foundations settlement, plan dimensions of 1.5 m x 1.5 m x 0.4 m for length, breadth and depth, respectively, were assumed. The variation of foundation settlement with depth of 200 kN/m² applied foundation pressure for CPT 3 is shown in Figure 2. Foundation elastic settlement decreased with depth having the highest values in the borehole designated as CPT 3.

Fig. 1. Flow chart for the reliability analysis.
The variation of CPT tip resistance as obtained from the field test results with penetration depth is shown in Figure 3. The least resistance values which are directly indicative of low strength and high compressibility of the construction site soil was observed in CPT 3 borehole. Since engineering design is normally based on the worst scenario, further analysis were all based on the CPT 3 results.

![Fig. 2. Variation of foundation settlement with embedment depth.](image1)

![Fig. 3. Variation of CPT tip resistance with penetration depth.](image2)
3.2. Reliability analysis

Reliability indices were calculated with the objective of developing a risk analysis procedure specifically for prediction of the settlement of foundations lying on soils. Reliability, as a measure of structural performance, was expressed in terms of reliability indices which were calculated for a total settlement of shallow foundations based on the Burland and Burbidge [27] settlement prediction method. Tolerable (allowable) settlement of 25 mm, as recommended by Eurocode 7, was considered and was treated as a deterministic value.

It was observed that, as the variability of geotechnical properties at a site increases (i.e., as the site becomes more heterogeneous), larger settlement values were obtained with a higher probability of occurrence. In Figures 4 - 8, as the coefficient of variation (COV) of the CPT tip resistance increases, there is an increase in the inherent variability of the site and/or the measurement error, the reliability index (β) of settlements decreased (and invariably, the associated probability of failure (Pf) increased). It implies that both the range and the maximum value of the expected settlement become larger. The success of a foundation design that estimates settlements from field test results depends on the uncertainty of the site geotechnical parameters. Using these predicted settlement values, without considering the qualities and uncertainties in the available test type, test results and design information can be misleading.

Fig. 4. Variation of safety index with foundation depth for 50 kN/m² applied pressure.
Fig. 5. Variation of safety index with foundation depth for 100 kN/m² applied pressure.

Fig. 6. Variation of safety index with foundation depth for 200 kN/m² applied pressure.

Fig. 7. Variation of safety index with foundation depth for 300 kN/m² applied pressure.
Fig. 8. Variation of safety index with foundation depth for 500 kN/m$^2$ applied pressure.

It was also observed that the probability of failure ($P_f$) decreases with increasing foundation embedment depth, increases with increasing COV of CPT tip resistance and increases with increase in applied foundation pressure. These observations are not unexpected. Firstly, a more restrictive $\beta$-value leads to decrease in the design value of the applied foundation stress. Secondly, an increase in the uncertainty of CPT tip resistance yields a less reliable compressibility value. Thirdly, as the footing embedment depth increases, the correlation between the compressibility characteristics of the soil beneath the footing decreases. These trends are in conformity with findings reported by Akbas and Kulhawy [21]. It should be noted that high value of safety (reliability) index (with reference to the target reliability index) implies that the structure is too safe and the consequence of this is a conservative design with high cost (uneconomical) while a lower value implies unsafe structure.

For applied foundation pressure not less than 300 kN/m$^2$, it was observed that the reliability indices at depth 1.2 m is either negative or very low thus indicating either certainty of failure or unreliable safety. Based on this observation, footings with an applied pressure greater than 300 kN/m$^2$ should either be embedded deeper than 1.2 m from the earth surface. The variability of the geotechnical parameters significantly affects the magnitude of settlements as shown in Figures 4 - 8. Therefore, the uncertainty in the design parameters should be considered for a more robust footing design procedure. This aim can be achieved systematically and consistently using the method of reliability based design (RBD).

3.3. Sensitivity study

A sensitivity study showed that the applied foundation pressure and COV of CPT tip resistance are very important for evaluating the magnitude of foundation settlements. Variation of reliability
index with foundation applied pressure is shown in Fig. 9. Similarly, typical values of reliability index ($\beta$) and the corresponding probability of failure ($P_f$) are given as a function of the allowable settlement for serviceability limit state (SLS) design of footings. It was observed that, as the inherent variability of the site geotechnical parameters increases, there is an increased probability of exceeding this allowable settlement value. This implies that site characteristics need serious consideration in a reliability-based design of foundations. It should be noted that the results of this study are particularly for the Burland and Burbidge [27] foundation settlement prediction method. The variation of safety index with applied foundation pressure for 1.8 m embedment only is shown in Fig. 9. For applied foundation pressure greater than 200 kN/m$^2$ based on the COV of CPT tip resistance of 26 and 30 %, it is recommended that deeper foundation embedment should be considered.

3.4. Target reliability levels
The acceptable safety levels can be expressed in terms of a target reliability index ($\beta_T$), which should be established for various design requirements. The selection of the target reliability level is a multi-specialty task involving a structural safety analysis and an economic analysis, which are the most two important factors. Generally, reliability indices below the target reliability value, $\beta_T$, are not acceptable. Target reliability indices calculated for settlement of foundations can vary with the suggested values of tolerable total and differential settlements and COV of CPT tip resistance. In other words, for each assumption of allowable settlement value, different values of target reliability indices can be obtained. Likewise, for each assumption of COV of CPT tip resistance, different values of target reliability indices can be obtained. AASHTO-LRFD
code [28] does not assume a specific value for allowed total settlement because it is only calibrated for strength limit states and is still not calibrated for serviceability limit states [21]. However, an allowable settlement value of 25 mm was recommended by Eurocode 7 for serviceability limit state.

The variation of the safety index with the settlement is shown in Fig. 10. The target reliability indices ($\beta_T$) based on the total allowable settlement of 25 mm for serviceability limit state (SLS) design of 14.36, 6.95, 5.03, 4.52 and 2.87 for 10, 20, 26, 30 and 40 % COV of CPT tip resistance, respectively, were recorded. The implication of using a lower value of COV of CPT tip resistance (with respect to the suggested 26 % COV value associated with Burland and Burbidge [27] method) for design is that the safety of the structure will be overestimated which is very risky and dangerous. On the other hand, using a higher value of COV of CPT tip resistance for design will lead to higher foundation size that if the economy is brought into consideration could result into the recommendation of a raft or deep foundation system instead (which is uneconomical). Circumstances that could warrant this condition include poor quality site investigation and highly variable geology.

![Fig. 10. Variation of safety index with the settlement.](image)

However, considering the manner in which CPT is mostly being carried out in the field which is mostly manually operated and not free from poor conduct of the test with a lot of negligence on several standard procedures, the use of COV value of 30 % of CPT tip resistance based on the Burland and Burbidge [27] method for SLS design, is highly recommended for RBD of footings total settlement on soils for the site. This COV value of 30 % of CPT tip resistance corresponds to target reliability index ($\beta_T$) of 4.52 and target probability of failure ($P_fT$) of 0.000677 %. This
P<sub>fT</sub> value seems satisfactory. Ahmed [21] reported a COV of SPT N-value of up to 48 % for automatic hammer SPT test and in a study on RBD of footings, Phoon et al. [5] estimated that the coefficient of variation (COV) of measurement error for SPT N-value was between 15 and 45 % for a range of cohesionless soils encountered during the field tests. Salahudeen et al. [18–20] reported a COV of SPT N-value of 30 %.

4. Conclusion

The development of a new generation of design codes that include reliability has been accepted as a rational measure of structural performance including geotechnical structures like foundations. Based on the results of the study carried out the following conclusions can be made:

1. The variability of the geotechnical parameters is highly influenced and has a significant effect on the settlement and safety of any structure.

2. The sensitivity study indicated that the applied foundation pressure and COV of CPT tip resistance significantly affected the magnitude of foundation settlements.

3. The target reliability indices (β<sub>T</sub>) based on the allowable total settlement of 25 mm for serviceability limit state (SLS) design of 14.36, 6.95, 5.03, 4.52 and 2.87 for 10, 20, 26, 30 and 40 % COV of CPT tip resistance, respectively, were recorded.

4. The methodology outlined and reliability output can be used as a basis for the establishment of RBD approach of footings in Nigeria and development of an LRFD specification.

5. The use of COV value of 30 % of CPT tip resistance based on the Burland and Burbidge method for SLS design is recommended for RBD of footings total settlement on soils. This COV value of 30 % of CPT tip resistance corresponds to the target reliability index (β<sub>T</sub>) of 4.52 and target probability of failure (P<sub>fT</sub>) of 0.000677% which is satisfactory.

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