Journal of Soft Computing in Civil Engineering 2-1 (2018) 01-17

journal homepage: http://www.jsoftcivil.com/

Fuzzy Vibration Suppression of a Smart Elastic Plate using Graphical Computing Environment

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ARTICLE INFO

Article history:
Received: 11 August 2017
Accepted: 28 September 2017

Keywords:
Smart plate model,
Spatial discretization,
Fuzzy control,
Computational algorithm,
SIMULINK diagrams.

ABSTRACT

A nonlinear model for the vibration suppression of a smart composite elastic plate using graphical representation involving fuzzy control is presented. The plate follows the von Kármán and Kirchhoff plate bending theories and the oscillations are caused by external transversal loading forces, which are applied directly on it. Two different control forces, one continuous and one located at discrete points, are considered. The mechanical model is spatially discretized by using the time spectral Galerkin and collocation methods. Our aim is to suppress vibrations through a simulation process within a modern graphical computing environment. Here we use MATLAB/SIMULINK, while other similar packages can be used as well. The nonlinear controller is designed, based on an application of a Mamdani-type fuzzy inference system. A computational algorithm, proposed and tested here is not only effective, but robust as well. Furthermore, all elements of the study can be replaced or extended, due to the flexibility of the used SIMULINK environment.

1. Introduction

Classical mathematical theories of control work well on linear systems. However, their effectiveness depends on many restrictions. On the other hand nonlinear controllers, based on fuzzy logic with built in smart computational methods can give a good description of a behaviour of nonlinear structures and provide a nonlinear feedback. The details of comparison between the classical control and fuzzy logic approach are discussed, among others, in the paper of Tairidis et al. [1].

An active vibration control of smart elastic plates has been considered by many authors. An active vibration control incorporating active piezoelectric actuator and self-learning control for a flexible plate structure is described by Tavakolpour et al. [2]. In the paper of Fisco and Adeli [3] hybrid control strategies, in particular active and semi-active vibration suppression are discussed. A brief survey on industrial applications of fuzzy control in different fields of engineering is given in the works of Precup and Hellendoorn [4]. The control systems are classified on three groups, control systems with Mamdani fuzzy controllers, control systems with Takagi-Sugeno fuzzy controllers, and adaptive and predictive control systems. A review of active structural control is done by Korkmaz in [5].

Here a vibration suppression of a smart plate is performed by the algorithm, involving constructed model-diagrams of SIMULINK with the use of Mamdani FIS. In the paper [6] a Newmark technique is applied for solving the system of linear ordinary differential equations (ODEs), obtained after the spatial discretization of the model. In this paper, we use a graphical representation for solving linear and nonlinear ODEs. The numerical simulation is implemented with the use of model-diagrams of SIMULINK. That allows easily and quickly suppressing vibrations of the plate and improves the presentation of the results. We can also include a Sugeno-type fuzzy inference system in the complete model-diagram for the linear model, i.e., the new approach provides flexibility of choosing a controller. Besides, we have tested a collocation method, which provides an opportunity to locate control forces at discrete points of the plate. The proposed algorithm allows effective suppression of the Fourier coefficients, which actually display characteristics of displacement, velocity and Airy's stress function (in other words, general displacement, velocity and Airy's function) of the plate. After computing the coefficients we can easily calculate the displacement and velocity at each point of the plate.

Active fuzzy control is a suitable tool for the systematic development of nonlinear control strategies and can be fine-tuned if no experience exists in the behaviour of considered body (structure) or if one designs more complicated control schemes, (e.g. [7-11]). Similar approaches for active vibration control of a simply supported rectangular plate using fuzzy logic rules are discussed in the paper of Shirazi et al. [11]. The results have been compared with the classical proportional integral derivative (PID) controller.

The present paper is organized as follows. Section 2 focuses on a formulation of the nonlinear dynamic mechanical model, and set up initial and boundary conditions. In Section 3 the spatially discretized model is presented. In both cases (Galerkin' projections and collocation approach) the result of discretization is a system of second order nonlinear ordinary differential equations of motion. In Section 4 an algorithm for vibration suppression is introduced. The algorithm involves, constructed model-diagrams with composed ``S-Function'' of SIMULINK. A numerical example using Galerkin's projections and the developed algorithm is presented. Section 5 is devoted to a state space representation of the linearized spatially discretized problem. We reformulate above mentioned algorithm for the linear model. Instead of the ``S-Function'' a ``State-Space'' block mode' is activated. The implementation of the collocation method for the linear case is illustrated in a numerical example. The main results are reported in Section 6.

2. Formulation of the Problem

Governing equations. A nonlinear mathematical model describing the bending vibrations of an isotropic, homogeneous elastic plate in the presence of active control, and allowance for the rotational inertia of the plate elements and viscous damping is written as

$$L_1(w,\psi) \equiv \rho h w_{tt} - \rho \frac{h^3}{12}\Delta w_{tt} + hcw_t + D\Delta^2 w - h[w,\psi] = [Q] + [Z], \quad (1)$$

$$L_2(w,\psi) \equiv \Delta^2\psi - \frac{E}{2}[w,w] = 0, \quad (t,x,y) \in \Omega. \quad (2)$$

Here the following notations are used:

$[w,\psi] = \partial_{11}w\partial_{22}\psi + \partial_{11}\psi\partial_{22}w - 2\partial_{12}w\partial_{12}\psi$ (Monge-Ampére's form).

- w is the deflection (displacement) of the plate.
- $\psi(t,x,y)$ is the Airy stress potential describing internal stresses, which appear due to the deformation of the plate (e.g., [12-14]).
- $\Omega = (0,T] \times G$, where T is the final time and $G = (0,l_1) \times (0,l_2)$ is the shape of the plate (l_1 and l_2 are the lengths of the sides of the plate).
- ρ is the density of the material.
- h is the thickness of the plate.
- c is the viscous damping coefficient.
- D is the flexural rigidity of the plate.
- $[Q]$ are the external transversal loading forces.
- $[Z]$ are the control forces.

The equations Eq.1, Eq.2 are extensions of the nonlinear von Kármán plate model for large deflections ([13], [14] etc.) on a control case. The plate is subjected to external transversal disturbances and generalized control forces, produced, for example, by electromechanical coupling effects.

For the Galerkin's projections it is required that $w,\psi \in C^2(0,T;W^{2,2}(G))$, ($W^{2,2}$ is the Sobolev space). For the collocation method it is required that $w,\psi \in C^2\big(0,T;C^2(G)\big) \cup C\big(0,T;C^4(G)\big)$.

Regarding the loading forces we consider $[Q]$ as a function of (t,x,y) or as a discrete function only of t, defined at some collocation points of the plate. We assume the same for the control forces $[Z]$. When $[Z]$ is supposed to be a function of (t,x,y), it is expanded into double Fourier's series and we use the time spectral Galerkin method for spatial discretization of L_1, L_2. The suppression of vibrations is done through the control of the Fourier coefficients. Here we also introduce a new approach, by considering a time-discrete type of control force $[Z]$, located at some discrete-collocation points of the plate, which may be different from the points where external forces are applied.

Initial and boundary conditions. For the displacement and velocity at the initial time we assume

$$w(0,x,y) = u(x,y), \quad w_t(0,x,y) = v(x,y) \text{ in } G,$$

where the functions u, v belong to the quadratically integrable function space $L_2(G)$.

Regarding the boundary conditions we consider simply supported, partially and totally clamped plates (see [15], [16]).

3. Spatial discretization of the problem Eq.1, Eq.2

3.1. Time spectral expansions.

An approximate analytical solution of Eq.1, Eq.2 in the form of partial sums of double Fourier's series with the time-dependent coefficients ([15]) reads

$$W_N(t,x,y) = \sum_{i,j=1}^{N} w_N^{ij}(t)\,\omega_{ij}(\mathrm{x},\mathrm{y}), \qquad W_N(t,x,y) = \sum_{i,j=1}^{N} w_{t,N}^{ij}(t)\,\omega_{ij}(\mathrm{x},\mathrm{y}), \tag{3}$$

$$\Psi_N(t,x,y) = \sum_{i,j=1}^{N} \psi_N^{ij}(t)\,\varphi_{ij}(\mathrm{x},\mathrm{y}), \qquad (t>0), \tag{4}$$

where the global basis functions ω_{ij} are chosen to match the boundary conditions [15]. For the initial conditions we have

$$W_N(0,x,y) = u_N(x,y) = \sum_{i,j=1}^{N} w_N^{ij}(0)\,\varphi_{\mathrm{ij}}(\mathrm{x},\mathrm{y}), \tag{5}$$

$$W_{t,N}(0,x,y) = v_N(x,y) = \sum_{i,j=1}^{N} w_{t,N}^{ij}(0)\,\sigma_{\mathrm{ij}}(\mathrm{x},\mathrm{y}), \tag{6}$$

where φ_{ij} and σ_{ij} are the bases. It is assumed $\varphi_{ij} = \sigma_{ij} = \omega_{ij}$. In case when $[Z]$ is a function of (t, x, y) we expand it into double Fourier series with the basic functions ω_{ij}, i.e.,

$$[Z] \equiv Z_N(t,x,y) = \sum_{i,j=1}^{N} z_N^{ij}(t)\,\omega_{ij}(x,y). \tag{7}$$

According to the two algorithms, introduced first for the linear case in [6], and extended for the nonlinear case in [16] the vibration suppression can be performed by means of the control function Z or the Fourier coefficients $z_N^{ij}(t)$. In the present paper we study the second case.

3.2. Galerkin's projections.

Let the external loading force $[Q]$ is a quadratically integrable function Q, i.e., $Q \in L_2(G)$ and $[Z]$ is defined as Eq.7. Introducing the inner product in L_2 - space and applying Galerkin's projections to Eq.1, Eq.2 yields

$$\langle L_1(W_N, \Psi_N), \omega_{mn}\rangle = \langle Q, \omega_{mn}\rangle + \langle Z_N, \omega_{mn}\rangle,$$

$$\langle L_2(W_N, \Psi_N), \omega_{mn}\rangle = 0, \qquad m,n = 1,2,\dots,N.$$

Hence

$$\mathbf{M}\ddot{\mathbf{w}}_N(t) + \mathbf{C}\dot{\mathbf{w}}_N(t) + \mathbf{K}_1\mathbf{w}_N(t) = \mathbf{A}_{1,N}\big(\mathbf{w}_N(t), \boldsymbol{\psi}_N(t)\big) + \mathbf{P}\mathbf{q}_N(t) + \mathbf{F}\mathbf{z}_N(t), \tag{8}$$

$$\mathbf{K}_2\mathbf{w}_N(t) = \mathbf{A}_{2,N}\big(\mathbf{w}_N(t), \mathbf{w}_N(t)\big), \tag{9}$$

where $\mathbf{w}_N(t)$, $\boldsymbol{\psi}_N(t)$ are the vectors with the components which are the Fourier coefficients for the displacement Eq.3, and the Airy stress function Eq.4, respectively. We call them characteristics of the displacement and the Airy stress function, respectively. Further, $\mathbf{M} = \rho h(\mathbf{H} + (h^2/12)\mathbf{B})$ is the mass matrix ($\mathbf{B}$ is the approximation of the Laplacian), $\mathbf{C}$ is the viscous damping matrix, $\mathbf{K}_1$ is the stiffness matrix. The matrices $\mathbf{K}_1$, $\mathbf{K}_2$ are the result of approximation of the biharmonic operator and $\mathbf{A}_{1,N}$, $\mathbf{A}_{2,N}$ are nonlinear approximations of the Monge-Ampère forms. Finally, $\mathbf{P}$ and $\mathbf{F}$ are the matrices and $\mathbf{q}_N$, $\mathbf{z}_N$ are the vectors, obtained after the approximations of the external exciting pressure and control forces, respectively. The operators $\mathbf{H}$, $\mathbf{B}$, $\mathbf{P}$ and $\mathbf{F}$ take different forms depending on the boundary conditions. The description of the above defined operators are given in [15]. Substituting Eq.9 into Eq.8 we obtain

$$\mathbf{M}\ddot{\mathbf{w}}_N + \mathbf{C}\dot{\mathbf{w}}_N + \mathbf{K}_1\mathbf{w}_N = \mathbf{A}_{1,N}\left(\mathbf{w}_N, \mathbf{K}_2^{-1}\mathbf{A}_{2,N}(\mathbf{w}_N, \mathbf{w}_N)\right) + \mathbf{P}\mathbf{q}_N + \mathbf{F}\mathbf{z}_N. \tag{10}$$

For the components from Eq.10 we have

$$(\mathbf{M}\ddot{\mathbf{w}}_N)_{mn} + (\mathbf{C}\dot{\mathbf{w}}_N)_{mn} + (\mathbf{K}_1\mathbf{w}_N)_{mn} = \left(\mathbf{A}_{1,N}\left(\mathbf{w}_N, \mathbf{K}_2^{-1}\mathbf{A}_{2,N}(\mathbf{w}_N, \mathbf{w}_N)\right)\right)_{mn}$$

$$+(\mathbf{P}\mathbf{q}_N)_{mn} + (\mathbf{F}\mathbf{z}_N)_{mn}, \quad m, n = 1,2, \dots, N\,. \tag{11}$$

For the initial conditions Eq.5, Eq.6 after applying Galerkin's projections we obtain

$$\begin{aligned}\big(\mathbf{U}\mathbf{w}_N(0)\big)_{mn} &= \langle u_N(x,y), \omega_{mn}\rangle,\\ \big(\mathbf{V}\mathbf{w}_N(0)\big)_{mn} &= \langle v_N(x,y), \omega_{mn}\rangle, \qquad m, n = 1,2, \dots, N.\end{aligned}$$

where u, v are described in [15].

If the inputs for the fuzzy controller are $w_N^{mn}(t)$ and $\dot{w}_N^{mn}(t)$ then the output is the control $z_{mn}(t)$ for these coefficients.

3.3. Collocation approach.

Let us now consider the collocation points (x_k, y_l), $\{(x_k, y_l),\ 0 < x_k < l_1,\ 0 < y_l < l_2,\ k, l = 1,2, \dots, N\}$ on the spatial domain G for the system Eq.1, Eq. 2. Suppose that $[Q]$ is the function of time defined at some or all collocation points. Similary let the control forces $[Z]$ be put at some collocation points, which may coincide with the points of application of the loading pressure. We find the solution of Eq.1, Eq.2 in the form Eq.3, Eq.4 when W_N and Ψ_N satisfy the equations Eq.1, Eq.2 at the collocation points, i.e.

$$L_1(W_N, \Psi_N)|_{(x=x_k, y=y_l)} = [Q]|_{(x=x_k, y=y_l)} + [Z]|_{(x=x_k, y=y_l)}, \tag{12}$$
$$L_2(W_N, \Psi_N)|_{(x=x_k, y=y_l)} = 0, \quad k, l = 1,2, \dots, N. \tag{13}$$

Supposing that control forces are located at every or some collocation points, from Eq.12 and Eq.13 we obtain a system of nonlinear ordinary differential equations of motion with respect to w_N^{mn} and ψ_N^{mn}, similar with the previous one Eq.10

$$\overline{\mathbf{M}}\ddot{\mathbf{w}}_N + \overline{\mathbf{C}}\dot{\mathbf{w}}_N + \overline{\mathbf{K}}_1\mathbf{w}_N = \overline{\mathbf{A}}_{1,N}\left(\mathbf{w}_N, \overline{\mathbf{K}}_2^{-1}\overline{\mathbf{A}}_{2,N}(\mathbf{w}_N, \mathbf{w}_N)\right) + \overline{\mathbf{q}} + \overline{\mathbf{z}}\,, \tag{14}$$

and similar to Eq.11

$$\left(\overline{\mathbf{M}}\ddot{\mathbf{w}}_N\right)_{kl} + \left(\overline{\mathbf{C}}\dot{\mathbf{w}}_N\right)_{kl} + \left(\overline{\mathbf{K}}_1\mathbf{w}_N\right)_{kl} = \left(\overline{\mathbf{A}}_{1,N}\left(\mathbf{w}_N, \overline{\mathbf{K}}_2^{-1}\overline{\mathbf{A}}_{2,N}(\mathbf{w}_N, \mathbf{w}_N)\right)\right)_{kl} + \overline{\mathbf{q}}_{kl} + \overline{\mathbf{z}}_{kl}\,,$$
$$k, l = 1,2, \dots, N, \tag{15}$$

where $\overline{\mathbf{M}}$ is the mass matrix, $\overline{\mathbf{C}}$ is the damping matrix and $\overline{\mathbf{K}}_1$ is the stiffness matrix. The elements of the matrices are determined through computations of the partial derivatives of W_N and Ψ_N (see Eq.3, Eq.4) with respect to the spatial variables x, y at the collocation points. The elements of the matrices $\overline{\mathbf{M}}$, $\overline{\mathbf{C}}$, $\overline{\mathbf{K}}_1$ and $\overline{\mathbf{K}}_2$ can be easily calculated. They are the values of the basic functions $\omega_{ij}(x_k, y_l)$. Analogously, the nonlinear parts are defined.

Furthermore, in Eq.14 $\overline{\mathbf{q}}$ is a vector with components $\overline{q}_{kl}$, the values of the time-discrete forces at some collocation points $(x_k, y_l),\ k, l = N_1, N_1 + 1, \dots, N_2$ and $\overline{\mathbf{z}}$ is the vector with components $\overline{z}_{kl}$, the values of the control forces at some collocation points $(x_k, y_l),\ k, l = M_1, M_1 + 1, \dots, M_2$. . The collocation points where the external loading forces are applied are called loading points and the points where we put/locate the control are called control points. In case $N_1 \equiv M_1 \leq N$ and $N_2 \equiv M_2 \leq N$ the loading points coincide with the control points. Obviously, we can also deal with free collocation points, where the external and control forces are absent. At these points the values of the external and control forces are supposed to be zero.

The advantage of the collocation method over the Galerkin's projections is that we do not need to take the inner products, and we can consider external loading disturbances at the discrete points and locate the control forces at the places of (all or some) collocation points in the proper way. Inversely, the collocation points can be considered at the best positions for the control, i.e. the collocation points are chosen in order to provide optimal suppressions of vibrations of the plate.

A disadvantage of the collocation method is that one may lose an accuracy of the approximate solution between the collocation points, and the obtained mass, damping and stiffness matrices are not so sparse as in case of Galerkin's projections.

For the initial conditions from Eq.5, Eq.6 we have

$$\sum_{i,j=1}^{N} w_N^{ij}(0)\omega_{ij}(x, y) = u_N(x_k, y_l), \qquad \sum_{i,j=1}^{N} w_{t,N}^{ij}(0)\omega_{ij}(x, y) = v_N(x_k, y_l),$$

3.4. State-space representation

Let us now suppose $\mathbf{v}_1 \equiv \mathbf{w}$, $\mathbf{v}_2 \equiv \dot{\mathbf{w}}$ (here and below the index N is omitted for the convenience). Then from Eq.10 we obtain

$$\dot{\mathbf{v}}_1 = \mathbf{v}_2$$

$$\mathbf{M}\mathbf{v}_2 + \mathbf{C}\mathbf{v}_2 + \mathbf{K}_1\mathbf{v}_1 = \mathbf{A}_{1,N}\left(\mathbf{v}_1, \mathbf{K}_2^{-1}\mathbf{A}_{2,N}(\mathbf{v}_1, \mathbf{v}_1)\right) + \mathbf{P}\mathbf{q}_N + \mathbf{F}\mathbf{z}_N$$

or

$$\dot{\mathbf{v}}_1 = \mathbf{v}_2,$$

$$\mathbf{v}_2 = \mathbf{M}^{-1}\left[-\mathbf{C}\mathbf{v}_2 - \mathbf{K}_1\mathbf{v}_1 + \mathbf{P}\mathbf{q}_N + \mathbf{F}\mathbf{z}_N + \mathbf{A}_{1,N}\left(\mathbf{v}_1, \mathbf{K}_2^{-1}\mathbf{A}_{2,N}(\mathbf{v}_1, \mathbf{v}_1)\right)\right]. \quad (16)$$

Analogously, for the model Eq.14

$$\dot{\mathbf{v}}_1 = \mathbf{v}_2,$$

$$\mathbf{v}_2 = \overline{\mathbf{M}}^{-1}\left[-\overline{\mathbf{C}}\mathbf{v}_2 - \overline{\mathbf{K}}_1\mathbf{v}_1 + \overline{\mathbf{q}} + \overline{\mathbf{z}} + \overline{\mathbf{A}}_{1,N}\left(\mathbf{v}_1, \overline{\mathbf{K}}_2^{-1}\overline{\mathbf{A}}_{2,N}(\mathbf{v}_1, \mathbf{v}_1)\right)\right]. \quad (17)$$

4. Algorithm of suppression of the Fourier coefficients and their derivatives

The use of model-diagrams of SIMULINK allows us quickly and effectively calculate the suppressed Fourier coefficients in the expansions Eq.3, Eq.4. After that one can easily calculate the suppressed vibrations at each point of the plate.

The decision of the controller is passed on the transformation (mapping) of input and output variables into membership functions along with a set of ``rules''. A total number of 15 rules are used here (Table 1). All rules have weight equal 1 and ``AND'' type logical operator is used.

Table 1. The fuzzy inference rules (e.g. if the displacement is ``FarUp'' and the velocity is ``UP'' then the control force is ``Max'')

Vel\Disp.	FarUp	CloseUp	Equil	CloseDn	FarDn
Up	**Max**	**Med+**	**Low+**	**Null**	**Low-**
Null	**Med+**	**Low+**	**Null**	**Low-**	**Med-**
Down	**Low+**	**Null**	**Low-**	**Med-**	**Min**

The Mamdani FIS has two inputs and one output. The inputs for the FIS are the displacement and velocity, and the output is the control force. Triangular and trapezoidal shape membership functions are chosen both for the inputs, Figures 1, 2, and for the output, Figure 3.

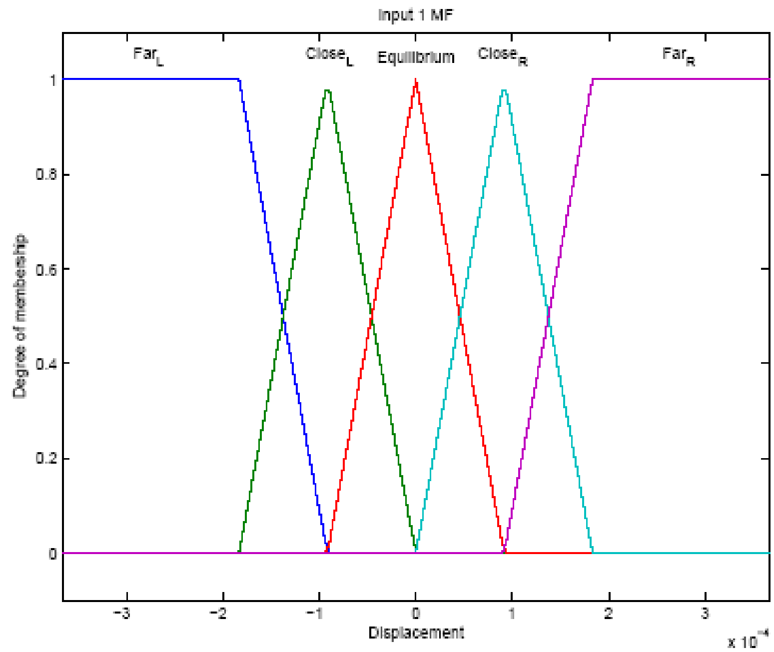

Fig. 1. Membership functions for the input (displacement) of the fuzzy inference system.

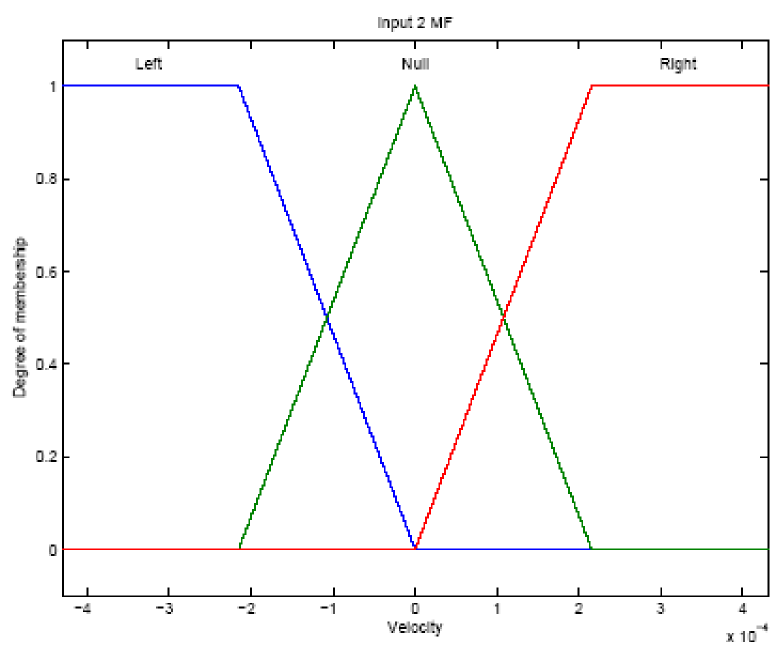

Fig. 2. Membership functions for the input (velocity) of the fuzzy inference system.

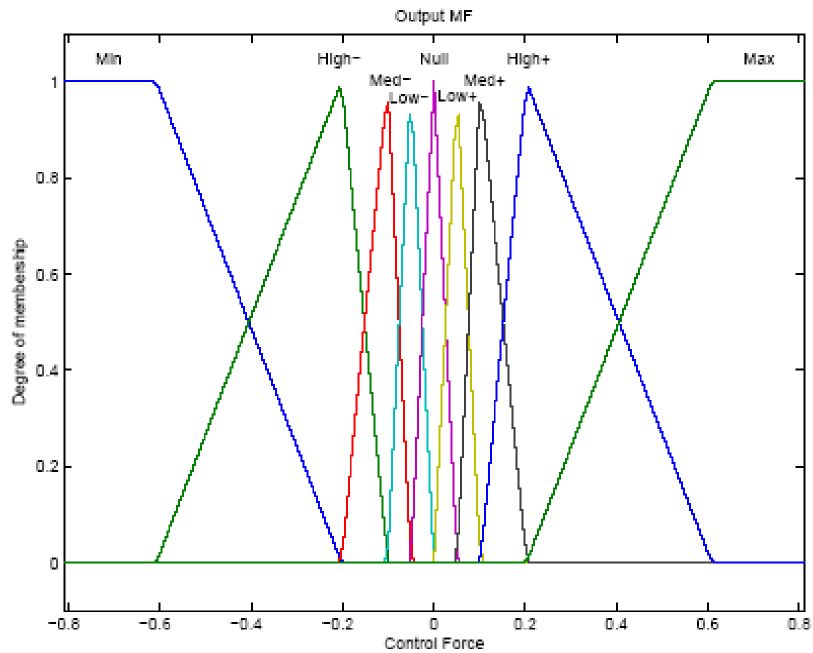

Fig. 3. Membership functions for the output (control forces) of the fuzzy inference system.

The implication method is set to minimum (min) and aggregation method is set to maximum (max). The defuzzified output value is created by using the Mean of Maximum defuzification method.

A trial-and-error adjustment is used for tuning the coefficients of the membership functions. A fully automatic fine-tuning using genetic and particle swarm optimization ([17], [18]) and also training the data with ANFIS ([19], [20]) is possible and very effective for particular cases.

Below a step by step algorithm for suppression of the Fourier coefficients in SIMULINK-MatLab environment with the use of a Mamdani type fuzzy inference system (FIS) is presented. For the nonlinear system two diagrams of SIMULINK, an initial one (Figure 4) and a complete model-diagram (Figure 5) are employed in the simulation. The diagrams include ``S-Function'' option which allows the insertion of MatLab code into a SIMULINK block. In Figures 4, 5 ``S-Function'' is the MatLab code *Sfun_NonLin*. The initial module is intended for computations of the coefficients without control and the complete module makes suppression of the coefficients. The suppression is performed by a nonlinear Mamdani fuzzy controller ([6], [16]).

In Figure 5 the suppression of the coefficients w^{11}, w_t^{11} in Eq.3 are provided on each time step of the simulation by the model-diagram. The control is put in the first equation in Eq.11 and Eq.15 for the first component, i.e. $z^{11} \neq 0$ and the others are zero. The control function Eq.7 is easily estimated at every point (x, y).

Algorithm for vibration suppression

Step 1. Set up input data of the initial diagram (Figure 4) in the block ``Sine Wave" by defining an amplitude, frequency phase etc. of exciting loading harmonic function $Q(t) = p\sin(\omega t + b) + \phi$. Here p is the amplitude, ω is the frequency, b is the bias and ϕ is the phase.

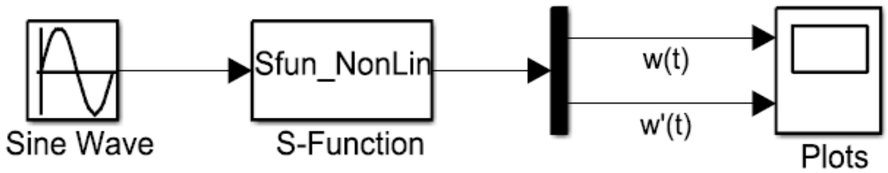

Fig. 4. The initial model-diagram for the nonlinear system

Step 2. Set up a number of basic functions N (formulas Eq.3, Eq.4). Correspondingly, determine the sizes of input and output data in the initialization part of ``S-Function", initialize the initial conditions for the system Eq.15 and the array of sample time.

Step 3. Run the initial module (without control) for obtaining a numerical solution of Eq.16 or Eq.17, i.e. in order to obtain the necessary data for the training of the controller. The output are the discrete values $w^{i_0 j_0}(t_k)$ and, $w_t^{i_0 j_0}(t_k)$ which are written as xi.

Step 4. Generate Mamdani-type fuzzy inference system (FIS) using the results obtained in Step 3, get the rules and then load the composed FIS on Workspace using Fuzzy Logic Toolbox: *fuzzy*.

Step 5. Run the complete module, Figure 5 (``Selector" block corresponds to Example 1), with the use of the obtained FIS in order to get suppressed $w^{i_0 j_0}(t_k)$ and $w_t^{i_0 j_0}(t_k)$. (We can use this FIS for computations of the other coefficients, $w^{ij}(t_k)$, $w_t^{ij}(t_k)$, $i \neq i_0, j \neq j_0$). The complete model-diagram incorporates two subsystems (``Subsystems 1, 2"). ``Subsystem 1" is intended for getting results without control and ``Subsystem 2" involves the control. The subsystems have the same structure with the initial model-diagram, presented in Figure 4.

Step 6. Switch on computing another coefficient, namely change the outputs in ``S-Function" and go to Step 3. Otherwise, go to Step 1 or Step 2 or terminate the simulation.

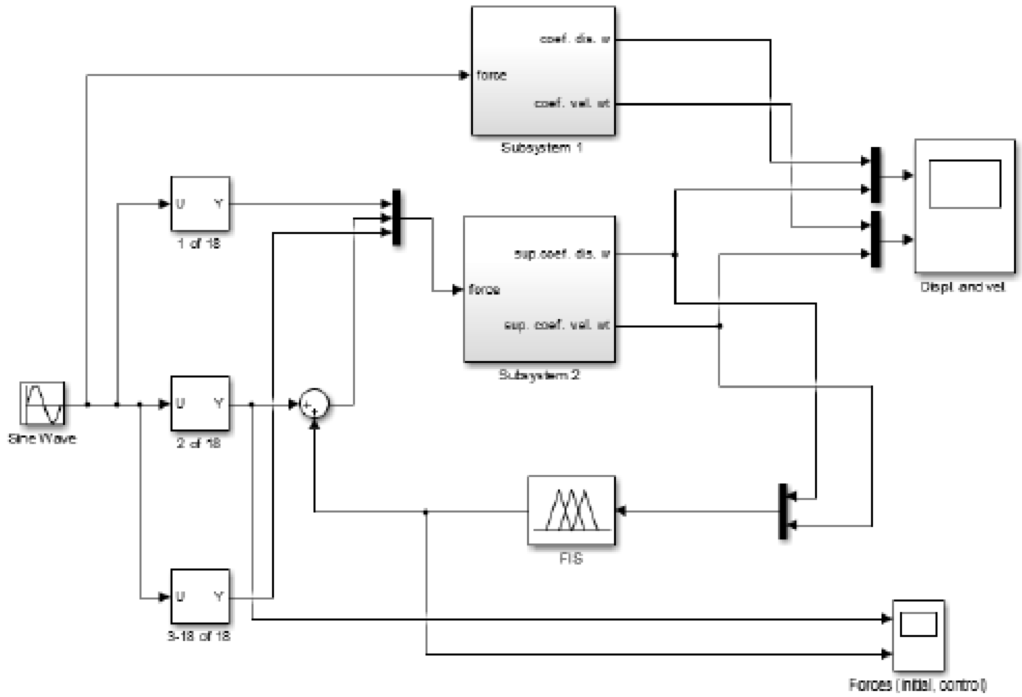

Fig. 5. The complete model-diagram.

In order to calculate the suppressed displacement $W(t_k, x, y)$ and suppressed velocity $W_t(t_k, x, y)$ at any point (x, y) of the plate at discrete time $t = t_k$ we use the values of the coefficients, obtained by the above given algorithm and substitute them into the analytical expressions Eq.3. The coefficients $\psi_N^{ij}(t)$ in Eq.4 can be computed through the equations Eq.9 or Eq.13. We can compute the coefficients as many as necessary in order to get more accurate solution in sense of suppression of the displacement $W(t, x, y)$ and velocity $W_t(t, x, y)$ at each point of the plate.

Below we consider an example of vibration suppressions of a simply supported plate by the described above algorithm with the use of the time spectral (Galerkin) method, Section 3.

Example 1.
Let the external force be a harmonic exciting load, $Q(t) = \sin 10\pi\, t$ uniformly distributed on the simply supported plate. Then $p_{mn} = (Q, \omega_{mn}) = (\mathbf{P}\,\mathbf{q}_N)_{mn} = 8\sqrt{l_1 l_1}/\pi^2 mn$. The amplitude in the option for the ``Sine Wave'' block in Figures, 4, 5 is a vector **P** with 18 components. It is defined as $\mathbf{P} = (0, p_{11}, 0, p_{12}, 0, p_{13}, 0, p_{21}, 0, p_{22}, 0, p_{23}, 0, p_{31}, 0, p_{32}, 0, p_{33})^T$. The control forces are added only to the second component p_{11}, i.e. to the right hand side of the second equation Eq.16. Thus, we have three ``Selector'' blocks: ``1 of 18'', ``2 of 18'' and ``3-18 of 18''. The control forces are added in the ``Selector'' block ``2 of 18'', i.e. to the second component of the vector **P**.

For the physical parameters of the plate we have $l_1 = l_2 = 1$, $E = 2$, $\nu = 0.3$, $h = 0.5$, $\rho = 1000$, $c = 0$, $D = 1$. Furthermore, the number of basic functions $N = 3$ and the final time $T = 30$. The *ODE solver* chooses a time step with respect to the given error, initial and minimal time steps. The first coefficients in Eq.3 are calculated. The results are presented in Figures 6 and 7 with the use of the Scope of SIMULINK. In Figure 6 we display the characteristics of the displacement and velocity, i.e. the first Fourier coefficients w^{11} and w_t^{11} before and after the control. In Figure 7 the external initial and control forces are shown, respectively. The x-axis (horizontal) denotes t - the time of simulation in seconds while the vertical axes denote the first Fourier coefficients in the expansions for the displacement and velocity, Eq.3.

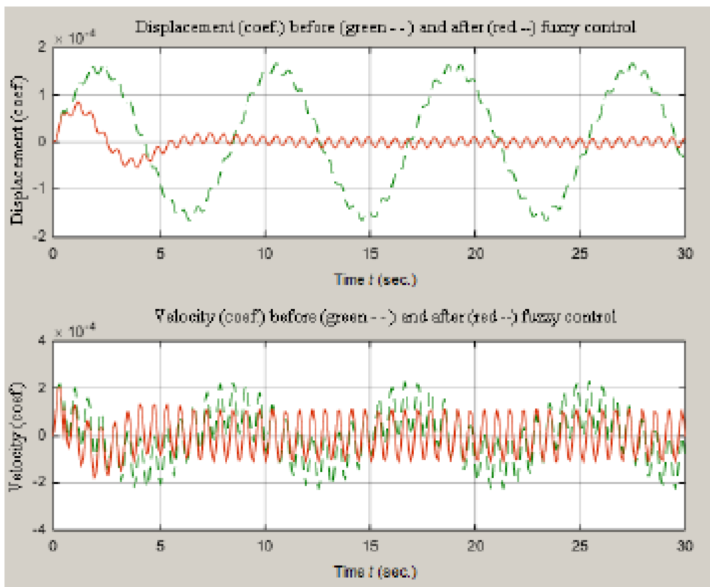

Fig. 6. Plots of the coefficients $w^{11}(t)$, $w_t^{11}(t)$ (characteristics of displacement and velocity) before and after the control.

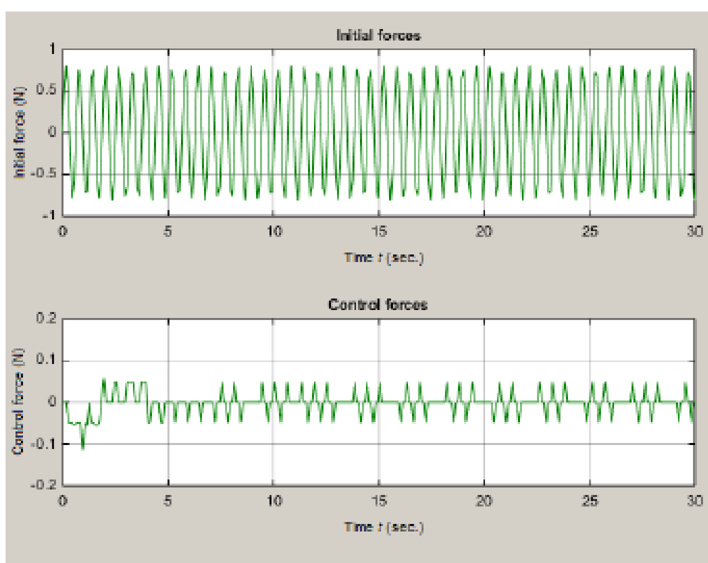

Fig. 7. The external and control forces with respect to time.

A detail analysis about estimates for the Fourier coefficients and the rate of convergence of the approximate solution are given in the work [15]. The time spectral as well the time collocation methods have a high accuracy order of computations, therefore, for small values of N we can obtain good results. Regarding the parameters of the exciting loading forces as many numerical experiments have shown changes of the frequency and the amplitude of the harmonic transversal loading in the neighborhood of them do not influence much on the control simulation. Therefore, ones we compose the FIS for the concrete values of the frequency and amplitude we can use this FIS in order to simulate the model for the frequency and amplitude close to the initial ones.

5. State-space representation for the linearized model

If we do not consider the nonlinear part in Eq.16 then the system Eq. 16 is a linear model which results a spartial discretetization of the Kirchhoff-Love plate model for small deflections ([11, 21, 22]). In this case we can use both ``S-Function'' and ``State-Space'' continuous block from the SIMULINK library.

Let us now write down the linearized model in the state space representation,

$$\begin{aligned} \mathbf{v} &= \tilde{\mathbf{A}}\mathbf{v} + \mathbf{B}\tilde{\mathbf{F}}, \\ \mathbf{w} &= \tilde{\mathbf{C}}\mathbf{v} + \mathbf{D}\tilde{\mathbf{F}}, \end{aligned} \tag{18}$$

where, $\mathbf{v} = \begin{bmatrix} \mathbf{v}_1 \\ \mathbf{v}_2 \end{bmatrix}$, $\tilde{\mathbf{A}} = \begin{bmatrix} \mathbf{0} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{bmatrix}$, $\mathbf{B} = \begin{bmatrix} \mathbf{0} \\ \mathbf{M}^{-1} \end{bmatrix}$, $\tilde{\mathbf{C}} = \begin{bmatrix} \mathbf{I} & \mathbf{0} \\ \mathbf{0} & \mathbf{I} \end{bmatrix}$, $\mathbf{D} = [\mathbf{0}]$ and $\tilde{\mathbf{F}} = \mathbf{P}\mathbf{q}_N + \mathbf{F}\mathbf{z}_N$. The size of the matrices $\tilde{\mathbf{A}}$, $\mathbf{B}$, $\tilde{\mathbf{C}}$ and $\mathbf{D}$ is $2 \times N^2 \times 2 \times N^2$. The identity matrix $\mathbf{I}$ and $\mathbf{M}, \mathbf{K}, \mathbf{C}$ have the size $N^2 \times N^2$. The vectors $\tilde{\mathbf{F}}$ and $\mathbf{v}$ have the size $2 \times N^2$.

We obtain analogous representations for the system Eq.17, if we neglect the nonlinear terms.

A similar algorithm for vibration supression of the Fourier coefficients for the linear model Eq. 18 is constructed. In this case, instead of "S-Function",' we employ ``State-Space'' representation from the SIMULINK library, which is a widely used tool in the classical control theory. In the algorithm for the linear problem in Step 2 we form the matrices for the state-space representation. Note, that in the block ``State-Space'', $\mathbf{A} = \tilde{\mathbf{A}}$, $\mathbf{C} = \tilde{\mathbf{C}}$ and $\mathbf{u} = \tilde{\mathbf{F}}$. The initial model-diagram for the linearized system is presented in Figure 8 (``Selector'' block corresponds to Example 2). The complete model-diagram with control uses the state-space representation inside its subsystems. It has a similar structure with the complete model-diagram, Figure 5. The "Subsystem 1" and "Subsystem 2" in this diagram are identical with Figure 8.

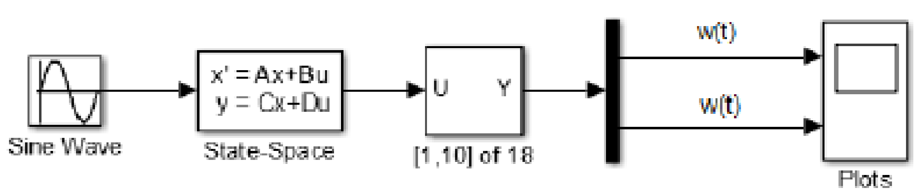

Fig. 8. The initial model-diagram for the linear system.

Example 2. Here we suppose that the external force $Q(t) = \sin 25\pi t$ is applied at some discrete points on the simply supported plate, exactly at the four collocation points: $(x_1, y_1) = (1/(N+1), 1/(N+1))$, $(x_1, y_3) = (1/(N+1), 3/(N+1))$, $(x_3, y_1) = (3/(N+1), 1/(N+1))$, $(x_3, y_3) = (3/(N+1), 3/(N+1))$. The physical parameters take the same values as in Example 1. The number of the basic functions $N = 3$ and the final time is $T = 45$. Here $\overline{q}_{11} = \overline{q}_{13} = \overline{q}_{31} = \overline{q}_{33} = \sin 25\pi t$ (see Eq. 15). The initial model-diagram is given in Figure 8 and the complete model-diagram has the same structure as in Figure 5. The amplitude in ``Sine Wave'' block is $\mathbf{P} = (0,0,0,0,0,0,0,0,0,1,0,1,0,0,0,1,0,1)^T$. The results are presented in Figures 9 and 10. The suppression of the first Fourier coefficients is shown. The x-axis (horizontal) denotes t - the time of simulation in seconds while the vertical axes denote the first Fourier coefficients in the expansions for the displacement and velocity, Eq.3.

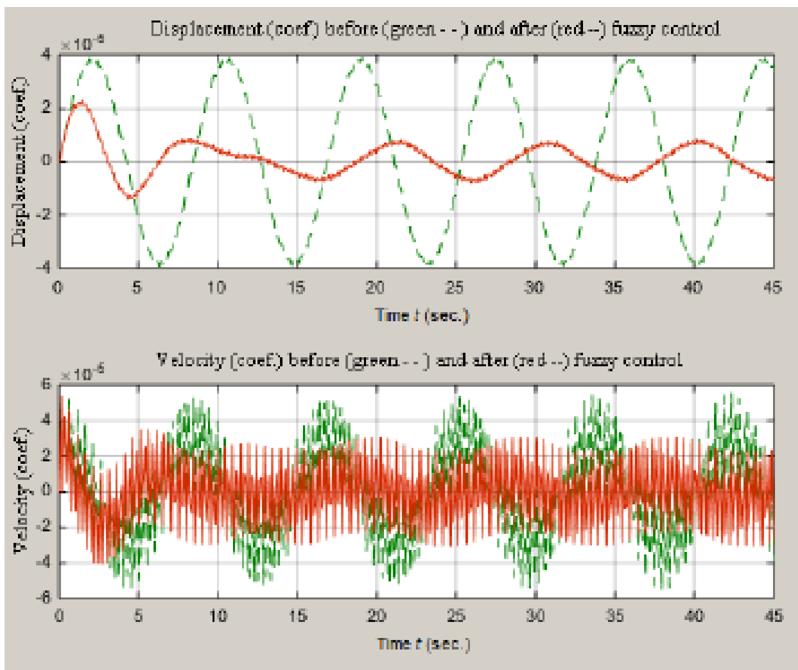

Fig. 9. Plots of the coefficients $w^{11}(t)$, $w_t^{11}(t)$ (characteristics of displacement and velocity) before and after the control

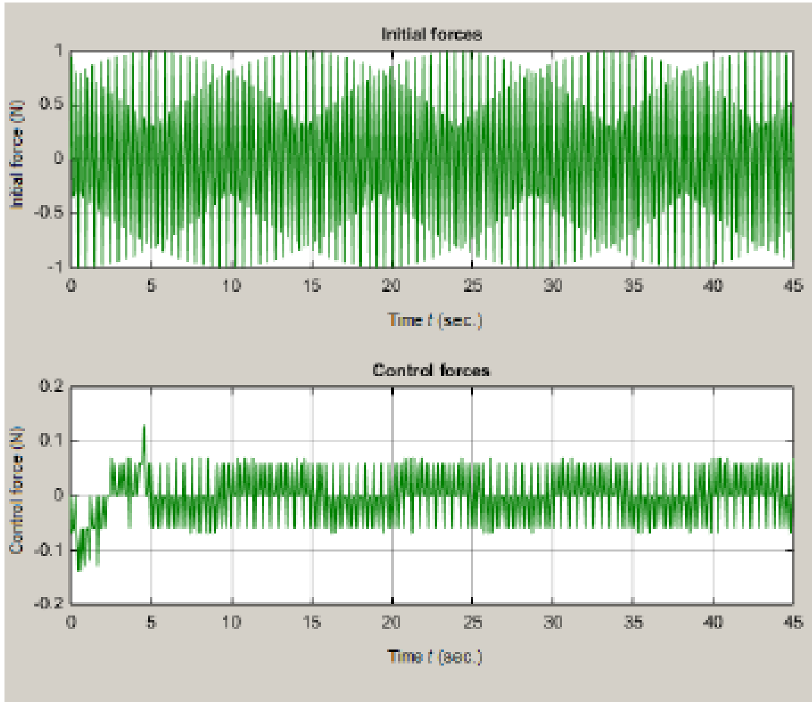

Fig. 10. The external and control forces with respect to time.

6. Conclusions

An effective algorithm for vibration suppression of a rectangular elastic plate with an application of SIMULINK-MATLAB, has been proposed. The mechanical plate model has been spatially discretized by two approaches: the time spectral (Galerkin) method, which was introduced before, and a new time spectral-collocation scheme, developed here. The presented algorithm involves model-diagrams, created with the use of SIMULINK library and Fuzzy ToolBox. A Mamdani type fuzzy inference system has been chosen.

The techniques have been demonstrated on two examples. Actually, we have shown suppression of the first Fourier coefficients, which characterize the behaviour of the plate. The time spectral (Galerkin) method provides more accurate computations, however the collocation method is more flexible in a sense of the location of control points.

Employing of SIMULINK allows quickly and effectively solving the studied control problems in linear and nonlinear mechanics and also to improve the presentation of the results. Several examples of suppression vibration of a plate with sinusoidal exciting loading forces can be easily implemented with the use of the proposed algorithm. The introduced methodology can be extended and applied to similar problems.

Acknowledgements

The second author has been supported by a postdoctoral research grant through the "IKY fellowship of excellence for postgraduate studies in Greece – Siemens programme".

REFERENCES

[1] Tairidis G. K., Stavroulakis G. E., Marinova D. G., & Zacharenakis E. C. Classical and Soft Robust Active Control of Smart Beams, In: Papadrakis, M., Charmpis, D. C. Lagaros, N. D., Tsompanakis, Y. (Eds), *Computat. Struct. Dynamics and Earthquake Engineer*, CRC Press/Balkema and Taylor & Francis Group, London, UK, 2009, Ch. 11, pp. 165–178.

[2] Tavakolpour A. R., Mailah M., Darus I. Z. M., & Tokhi O. Selflearning active vibration vibration control of a flexible plate structure with piezoelectric actuator. *Simul. Model. Prac. and Theory* **18,** 2010, 516–532.

[3] Fisco N.R. & Adeli H. Smart structures: Part II: Hybrid control systems and control control strategies. *Scientia Iranica* **18**, 2011, 285–295.

[4] Precup R.-E., & Hellendoorn H. A survey on industrial applications of fuzzy control. *Computers in Industry* **62**, 2011, 213–226.

[5] Korkmaz S. A review of active structural control: challenges for engineering informatics. *Comput. and Struct.* **89**, 2011, 2113–2132.

[6] Preumont A. Vibration Control of Active Structures, Springer, 2002.

[7] Driankov D., Hellendoorn H., & Reinfrank M. An Introduction to Fuzzy Control, 2nd ed., Springer-Verlag, Berlin, Heidelberg, New York, 1996.

[8] Zeinoun I. J., & Khorrami I. J. An Adaptive Control Scheme Based on Fuzzy Logic and its Application to Smart Structures. *Smart Mater. Struct.* **3**, 1994, 266–276.

[9] Wenzhonga Q., Jincaib S., & Yangc Q. Active control of vibration using a fuzzy control method. *J. of Sound and Vibration*, **275**, 2004, 917–930.

[10] Shirazi A. H. N., Owji H. R., & Rafeeyan M. Active vibration control of an FGM rectangular plate using fuzzy logic controllers. *Procedia Engineering* **14,** 2011, 3019–3026.

[11] Ciarlet Ph. G. Mathematical Elasticity, v. II: Theory of Plates. Elsevier, Amsterdam, 1997.

[12] Ciarlet P., & Rabier P. Les equations de von Kármán. Springer-Verlag, Berlin, Heidelberg, New York, 1980.

[13] Duvaut G., & Lions J. L. (1972). Les inequations en mecaniques et en physiques, Dunod.

[14] Muradova A. D. A time spectral method for solving the nonlinear dynamic equations of a rectangular elastic plate. *J. Eng. Math.* **92**, 2015, 83–101.

[15] Muradova A. D. & Stavroulakis G. E. Hybrid control of vibrations of smart von Kármán plate. *Acta Mechanica* **226**, 2015, 3463–3475.

[16] Muradova A. D., & Stavroulakis G. E. Fuzzy vibration control of a smart plate. *Int. J. Comput. Meth. in Eng. Sci. Mech.* **14,** 2013, 212–220.

[17] Tairidis G. Foutsitzi G., Koutsianitis P., Stavroulakis G.E. Fine tuning of a fuzzy controller for vibration suppression of smart plates using genetic algorithms. *Advances in Engineering Software*, **101**, 2016, 123–135.

[18] Marinaki M., Marinakis Y., Stavroulakis G. T. Fuzzy control optimized by a Multi-Objective Particle Swarm Optimization algorithm for vibration suppression of smart structures. *Structural and Multidisciplinary Optimization*, **43**, 2011, 29–42.

[19] Koutsianitis P., Tairidis G. K., Drosopoulos G. A., Foutsitzi G. A., Stavroulakis G. E. (2017). Effectiveness of optimized fuzzy controllers on partially delaminated piezocomposites, *Acta Mechanica*, **228**, 1373–1392.

[20] Muradova A. D., Tairidis G. K., Stavroulakis G. T. Adaptive Neuro-Fuzzy Vibration Control of a Smart Plate. *Numerical Algebra, Control and Optimization*, **7**, 2017, 251–271.

[21] Destuynder Ph., & Salaun M.. Mathematical Analysis of Thin Plate Models., Springer, 1996.

[22] Reddy J. N. Theory and Analysis of Elastic Plates and Shells, CRC Press, 2007, Taylor & Francis.