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Flexural Analysis of Deep Aluminum Beam

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ABSTRACT

Many parts of spacecrafts, airplane are made up of aluminum, which are thick or deep in section. For the analysis of deep or thick beams, a trigonometric shear deformation theory is used, taking into account transverse shear deformation effects, is developed. To represent the shear deformation effects, a sinusoidal function is used in displacement field in terms of thickness coordinate. The important feature of this theory is that the transverse shear stresses can be obtained directly from the use of constitutive relations with excellent accuracy, satisfying the shear stress conditions on the end surfaces of the beam. Hence, the theory obviates the need of shear correction factor. Using the principle of virtual work governing differential equations and boundary conditions are obtained. The thick aluminum beam is considered for the numerical study to show the accuracy of the theory. The cantilever beam subjected to cosine loads is examined using the present theory. Results obtained are discussed with those of other theories.

1. Introduction

Euler-Bernoulli hypothesis disregards the effects of the shear deformation and stress concentration which is in elementary theory of beam bending hence it is suitable for thin beams and is not suitable for deep beams since it is based on the assumption that the transverse normal to neutral axis remains so during bending and after bending, implying that the transverse shear strain is zero. Since theory neglects the transverse shear deformation. It underestimates deflections in case of thick beams where shear deformation effects are significant.

Timoshenko [1] showed that the effect of transverse vibration of prismatic bars. This theory is now widely referred to as Timoshenko beam theory or first order shear deformation theory (FSDT) in the literature. But in this theory transverse shear strain distribution is assumed to be constant through the thickness of beam and thus requires shear correction factor to appropriately represent the strain energy of deformation.

Cowper [2] has given refined expression for the shear correction factor for different cross-sections of beam. The accuracy of Timoshenko beam theory for transverse vibrations of simply supported beam in respect of the fundamental frequency is verified by Cowper [3] with a plane stress exact elasticity solution.

To remove the discrepancies in classical and first order shear deformation theories, higher order or refined shear deformation theories were developed and available in the open literature for static and vibration analysis of beam. Krishna Murthy [4], Baluch *et al*. [5], Bhimaraddi and Chandrashekhara [6] were presented parabolic shear deformation theories assuming a higher variation of axial displacement in terms of thickness coordinate. These theories satisfy shear stress free boundary conditions on top and bottom surfaces of beam and thus obviate the need of shear correction factor.

Kant and Gupta [7], and Heyliger and Reddy [8] presented finite element models based on higher order shear deformation uniform rectangular beams. However, these displacement based finite element models are not free from phenomenon of shear locking [9, 10].

Dahake and Ghugal [11] studied flexural analysis of thick simply supported beam using trigonometric shear deformation theory. Ghugal and Dahake [12, 13] given the flexural solution for the beam subjected to parabolic loading. Sawant and Dahake [14] developed the new hyperbolic shear deformation theory. Chavan and Dahake [15, 16] presented clamped-clamped beam using hyperbolic shear deformation theory. The displacement and stresses for thick beam given by Nimbalkar and Dahake [17].

Jadhav and Dahake [18] presented bending analysis of deep cantilever beam using steel as material. Manal et al [19] investigated the deep fixed beams using new displacement field. Patil and Dahake [20] carried out finite element analysis using 2D plane stress elements for thick beam. Dahake et al [21] studied flexural analysis of thick fixed beam subjected to cosine load. Tupe et al [22] compared various displacement fields for static analysis of thick isotropic beams.

In literature, most of the researchers have used steel as a beam material. As many parts of the spacecrafts, airplane structures are made up of aluminum due to its low weight density. In this research, an attempt has been made to analyze the aluminum deep cantilever beam subjected to cosine load.

2. Development of Theory

The beam under consideration occupies in $0-x-y-z$ Cartesian coordinate system the region:

$$0 \le x \le L \;; \quad 0 \le y \le b \;; \quad -\frac{h}{2} \le z \le \frac{h}{2}$$

where x, y, z are Cartesian coordinates, L and b are the length and width of beam in the x and y directions respectively, and h is the thickness of the beam in the z-direction. The beam is made up of homogeneous, linearly elastic isotropic material.

2.1. The displacement field

The displacement field of the present beam theory is of the form as given below:

$$u(x,z) = -z\frac{dw}{dx} + \frac{h}{\pi}\sin\frac{\pi z}{h}\phi(x)$$
$$w(x,z) = w(x) \quad (1)$$

where u is the axial displacement in x direction and w is the transverse displacement in z direction of the beam. The sinusoidal function is assigned according to the shear stress distribution through the thickness of the beam. The ϕ represents rotation of the beam at neutral axis, which is an unknown function to be determined.

Normal strain

$$\varepsilon_x = \frac{\partial u}{\partial x} = -z\frac{d^2 w}{dx^2} + \frac{h}{\pi}\sin\frac{\pi z}{h}\frac{d\phi}{dx} \quad (2)$$

Shear strain

$$\gamma_{zx} = \frac{\partial u}{\partial z} + \frac{dw}{dx} = \cos\frac{\pi z}{h}\phi \quad (3)$$

Stress-Strain Relationships

$$\sigma_x = E\varepsilon_x$$
$$\tau_{zx} = G\gamma_{zx} \quad (4)$$

2.2. Governing Equations and Boundary Conditions

Using the expressions for strains and stresses (2) through (4) and using the principle of virtual work, variationally consistent governing differential equations and boundary conditions for the beam under consideration can be obtained. The principle of virtual work when applied to the beam leads to:

$$b\int_{x=0}^{x=L}\int_{z=-h/2}^{z=+h/2}\left(\sigma_x \delta\varepsilon_x + \tau_{zx}\delta\gamma_{zx}\right)dx\,dz - \int_{x=0}^{x=L} q(x)\,\delta w\,dx = 0 \quad (5)$$

where the symbol δ denotes the variational operator. Employing Green's theorem in Eqn. (4) successively, we obtain the coupled Euler-Lagrange equations which are the governing differential equations and associated boundary conditions of the beam. The governing differential equations obtained are as follows:

$$EI\ \frac{d^4w}{dx^4} - \frac{24}{\pi^3}\ EI\ \frac{d^3\phi}{dx^3} = q(x) \tag{6}$$

$$\frac{24}{\pi^3}\ EI\ \frac{d^3w}{dx^3} - \frac{6}{\pi^2}\ EI\ \frac{d^2\phi}{dx^2} + \frac{GA}{2}\phi = 0 \tag{7}$$

The associated consistent natural boundary conditions obtained are of following form:

At the ends $x = 0$ and $x = L$

$$EI\frac{d^2w}{dx^2} = EI\frac{d\phi}{dx} = w = 0 \tag{8}$$

Thus the boundary value problem of the beam bending is given by the above variationally consistent governing differential equations and boundary conditions.

2.3. The General Solution of Governing Equilibrium Equations of the Beam

The general solution for transverse displacement $w(x)$ and warping function $\phi(x)$ is obtained using Eqns. (6) and (7) using method of solution of linear differential equations with constant coefficients. Integrating and rearranging the first governing Eqn. (6), we obtain the following equation

$$\frac{d^3w}{dx^3} = \frac{24}{\pi^3}\frac{d^2\phi}{dx^2} + \frac{Q(x)}{EI} \tag{9}$$

where $Q(x)$ is the generalized shear force for beam and it is given by $Q(x) = \int_0^x q\,dx + C_1$.

Now second governing Eqn. (7) is rearranged in the following form:

$$\frac{d^3w}{dx^3} = \frac{\pi}{4}\frac{d^2\phi}{dx^2} - \beta\phi \tag{10}$$

A single equation in terms of ϕ is now obtained using Eqns (11) and (12) as:

$$\frac{d^2\phi}{dx^2} - \lambda^2\phi = \frac{Q(x)}{\alpha EI} \tag{11}$$

where constants α , β and λ in Eqns. (10) and (11) are as follows

$$\alpha = \left(\frac{\pi}{4} - \frac{24}{\pi^3}\right),\ \beta = \left(\frac{\pi^3}{48}\frac{GA}{EI}\right) \text{and } \lambda^2 = \frac{\beta}{\alpha}$$

The general solution of Eqn. (11) is as follows:

$$\phi(x) = C_2\cosh\lambda x + C_3\sinh\lambda x - \frac{Q(x)}{\beta EI} \tag{12}$$

The equation of transverse displacement $w(x)$ is obtained by substituting the expression of $\phi(x)$ in Eqn. (12) and then integrating it thrice with respect to x. The general solution for $w(x)$ is obtained as follows:

$$EI\, w(x) = \iiiint q\, dx\, dx\, dx\, dx + \frac{C_1 x^3}{6} + \left(\frac{\pi}{4}\lambda^2 - \beta\right)\frac{EI}{\lambda^3}\left(C_2 \sinh\lambda x + C_3 \cosh\lambda x\right) + C_4 \frac{x^2}{2} + C_5 x + C_6 \quad (13)$$

where, C_1, C_2, C_3, C_4, C_5 and C_6 are arbitrary constants and can be obtained by imposing natural (forced) and / or geometric or kinematical boundary / end conditions of beam.

3. Illustrative Example

In order to prove the efficacy of the present theory, a numerical example is considered. For the static flexural analysis, a uniform beam of rectangular cross section, having span length '*L*', width '*b*' and thickness '*h*' of homogeneous, elastic and isotropic material is considered. The following material properties for beam are used.

Table 1. Properties of Aluminum 6061-T6, 6061-T651 [13]

Physical Properties	**Quantity**
Density	2700 kg/m^3
Ultimate Tensile Strength	310 MPa
Modulus of Elasticity	68.9 GPa
Notched Tensile Strength	324 MPa
Ultimate Bearing Strength	607 MPa
Bearing Yield Strength	386 MPa
Poisson's Ratio	0.33
Fatigue Strength	96.5 MPa
Shear Modulus	26 GPa
Shear Strength	207 MPa

3.1. Example: Cantilever Beam with Cosine Load

The beam has its origin at left hand side fixed support at $x = 0$ and free at $x = L$. The beam is subjected to cosine load, $q(x) = q_0 \cos\dfrac{\pi x}{2L}$ on surface $z = +h/2$ acting in the downward z direction with maximum intensity of load q_0.

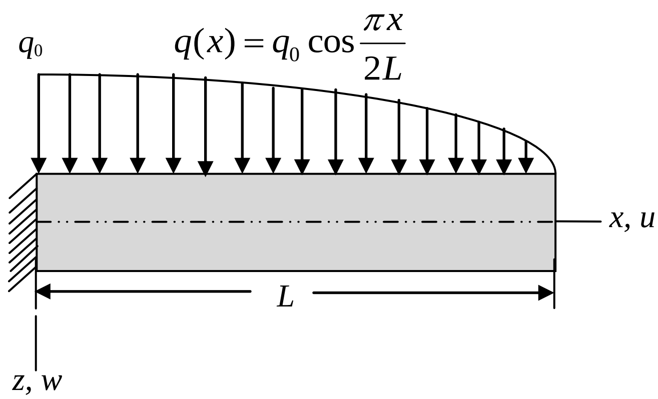

Fig. 2. Cantilever beam with cosine load

Boundary conditions associated with this problem are as follows:

At Free end: $EI\frac{d^2w}{dx^2} = EI\frac{d\phi}{dx} = EI\frac{d^3w}{dx^3} = EI\frac{d^2\phi}{dx^2} = 0$ at $x = L$ and

At Fixed end: $\frac{dw}{dx} = \phi = w = 0$ at $x = 0$

General expressions obtained for $w(x)$ and $\phi(x)$ are as follows:

$$w(x) = \frac{q_0 L^4}{120EI}\left\{\begin{aligned} & -\frac{960}{\pi^3}\left[-\frac{2}{\pi}\left(\cos\frac{\pi x}{2L} - 1\right) - \frac{1}{2}\frac{x^2}{L^2}\right] - \frac{4920}{\pi^4}\left(\frac{1}{6}\frac{x^3}{L^3} - \frac{1}{4}\frac{x^2}{L^2}\right) \\ & -\frac{240}{\pi^3}\frac{E}{G}\frac{h^2}{L^2}\left[-\frac{2}{\pi}\left(\cos\frac{\pi x}{2L} - 1\right) - \frac{1}{2}\frac{x^2}{L^2}\right] \\ & -5\frac{E}{G}\frac{h^2}{L^2}\left(-\frac{x}{L} - \frac{1}{2}\frac{x^2}{L^2} + \frac{\sinh\lambda x - \cosh\lambda x + 1}{\lambda L}\right) \end{aligned}\right\} \tag{14}$$

$$\phi(x) = \frac{q_0 L}{\beta EI}\left(1 - \frac{2\pi^3}{41}\sin\frac{\pi x}{2L} + \sinh\lambda x - \cosh\lambda x\right) \tag{15}$$

The axial displacement and stresses obtained based on above solutions are as follows

$$u=\frac{q_0 h}{Eb}\left[\begin{array}{l}-\frac{1}{10}\frac{z}{h}\frac{L^3}{h^3}\left(\begin{array}{l}-31\left(\sin\frac{\pi x}{2L}-\frac{x}{L}\right)-8\frac{E}{G}\frac{h^2}{L^2}\left(\sin\frac{\pi x}{2L}-\frac{x}{L}\right)\\ -5\frac{E}{G}\frac{h^2}{L^2}\left(\frac{x}{L}-1+\cosh\lambda x-\sinh\lambda x\right)-51\left(\frac{1}{2}\frac{x^2}{L^2}-\frac{1}{2}\frac{x}{L}\right)\end{array}\right)\\ -\frac{104}{5\pi^4}\sin\frac{\pi z}{h}\frac{E}{G}\frac{L}{h}\left(\frac{30}{13}\frac{x}{L}-\frac{10}{13}\frac{x^3}{L^3}-1+\cosh\lambda x-\sinh\lambda x\right)\end{array}\right] \tag{16}$$

$$\sigma_x=\frac{q_0}{b}\left[\begin{array}{l}-\frac{1}{10}\frac{z}{h}\frac{L^2}{h^2}\left(\begin{array}{l}60\frac{x^2}{L^2}-10\frac{x^4}{L^4}+8-52\frac{x}{L}-\frac{12}{\pi^2}\frac{E}{G}\frac{h^2}{L^2}\left(-\frac{x^2}{L^2}+\frac{1}{3}\right)\\ -\frac{26}{5}\frac{E}{G}\frac{h^2}{L^2}\left(1+\lambda L(\sinh\lambda x-\cosh\lambda x)\right)\end{array}\right)\\ -\frac{104}{5\pi^4}\sin\frac{\pi z}{h}\frac{E}{G}\left(\frac{30}{13}-\frac{30}{13}\frac{x^2}{L^2}+\lambda L(\sinh\lambda x-\cosh\lambda x)\right)\end{array}\right] \tag{17}$$

$$\tau_{zx}^{CR}=\frac{104}{5\pi^3}\frac{q_0}{b}\frac{L}{h}\cos\frac{\pi z}{h}\left(1+\frac{10}{13}\frac{x^3}{L^3}-\frac{30}{13}\frac{x}{L}+\sinh\lambda x-\cosh\lambda x\right) \tag{18}$$

$$\begin{aligned}\tau_{zx}^{EE}=&\frac{q_0 L}{80bh}\left(4\frac{z^2}{h^2}-1\right)\left[40\frac{x^3}{L^3}-4-\frac{240}{\pi^2}\frac{x}{L}-\frac{4}{5}\frac{E}{G}\frac{h^2}{L^2}\lambda^2L^2\left(\cosh\lambda x-\sinh\lambda x\right)\right]\\ &-\frac{16}{5\pi^5}\cos\frac{\pi z}{h}\frac{E}{G}\frac{q_0 h}{bL}\left[-1+5\frac{x^3}{L^3}+\lambda^2L^2\left(\cosh\lambda x-\sinh\lambda x\right)\right]\end{aligned} \tag{19}$$

4. Results

4.1. Numerical Results

In this paper, the results for inplane displacement, transverse displacement, inplane and transverse stresses are presented in the following non dimensional form for the purpose of presenting the results in this work.

For beam subjected to cosine load

$$\bar{u}=\frac{Ebu}{q_0 h},\quad \bar{w}=\frac{10Ebh^3w}{q_0L^4},\quad \bar{\sigma}_x=\frac{b\sigma_x}{q_0},\quad \bar{\tau}_{zx}=\frac{b\tau_{zx}}{q_0}$$

The transverse shear stresses ($\bar{\tau}_{zx}$) are obtained directly by constitutive relation and, alternatively, by integration of equilibrium equation of two dimensional elasticity and are denoted by ($\bar{\tau}_{zx}^{CR}$) and ($\bar{\tau}_{zx}^{EE}$) respectively. The transverse shear stress satisfies the stress free boundary conditions on the top $(z=-h/2)$ and bottom $(z=+h/2)$ surfaces of the beam when these stresses are obtained by both the above mentioned approaches.

Table 2: Non-Dimensional Axial Displacement ($\bar{u}$) at (x = L, z = h/2), Transverse Deflection ($\bar{w}$) at (x = L, z =0.0) Axial Stress ($\bar{\sigma}_x$) at (x = 0, z = h/2) Maximum Transverse Shear Stresses $\bar{\tau}_{zx}^{CR}$ (x=0.01L, z =0.0) and $\bar{\tau}_{zx}^{EE}$ (x, z = 0.0) of the Cantilever Beam Subjected to Cosine Load for Aspect Ratio 4 and 10.

Source	Aspect ratio	Model	$\bar{u}$	$\bar{w}$	$\bar{\sigma}_x$	$\bar{\tau}_{zx}^{CR}$	$\bar{\tau}_{zx}^{EE}$
Present		TSDT	-67.5989	6.1819	36.7529	1.8181	-2.7877
Sawant and Dahake [14]		HPSDT	-70.024	6.1928	39.8104	2.1609	-4.5581
Krishna Murty [4]	4	HSDT	-71.2291	6.1860	37.2887	1.9004	-2.8916
Timoshenko [1]		FSDT	23.1543	6.5444	22.2081	0.3076	3.7597
Bernoulli-Euler		ETB	23.1543	5.7541	22.2081	—	3.7597
Present		TSDT	-1055.4548	5.8244	176.7877	7.7501	3.2052
Sawant and Dahake [14]		HPSDT	-1061.5175	5.8256	178.4137	8.3023	3.8042
Krishna Murty [4]	10	HSDT	-1064.5122	5.8255	172.1017	7.8208	3.7523
Timoshenko [1]		FSDT	361.7856	5.8805	138.8010	4.8073	9.3993
Bernoulli-Euler		ETB	361.7856	5.7541	138.8010	—	9.3993

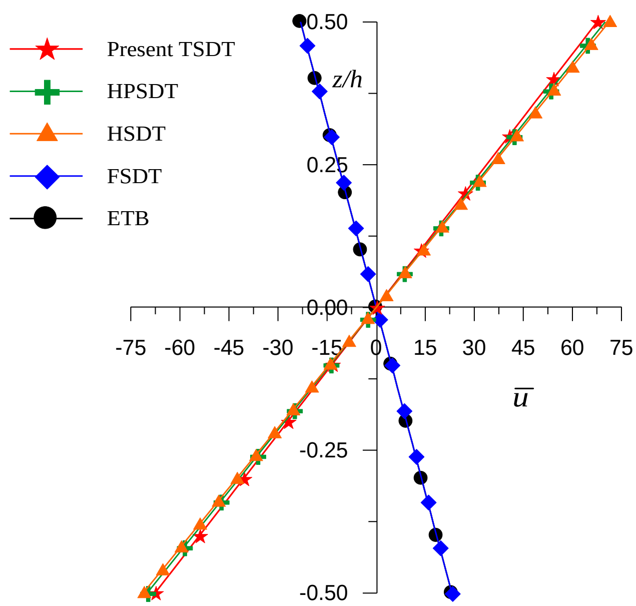

Fig. 2. Variation of axial displacement ($\bar{u}$) through the thickness of cantilever beam at (x = L, z) for aspect ratio 4.

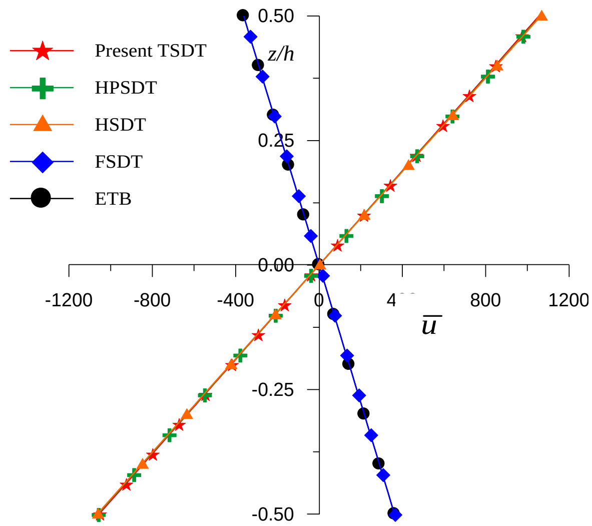

Fig. 3. Variation of axial displacement ($\bar{u}$) through the thickness of cantilever beam at ($x = L$, z) for aspect ratio 10.

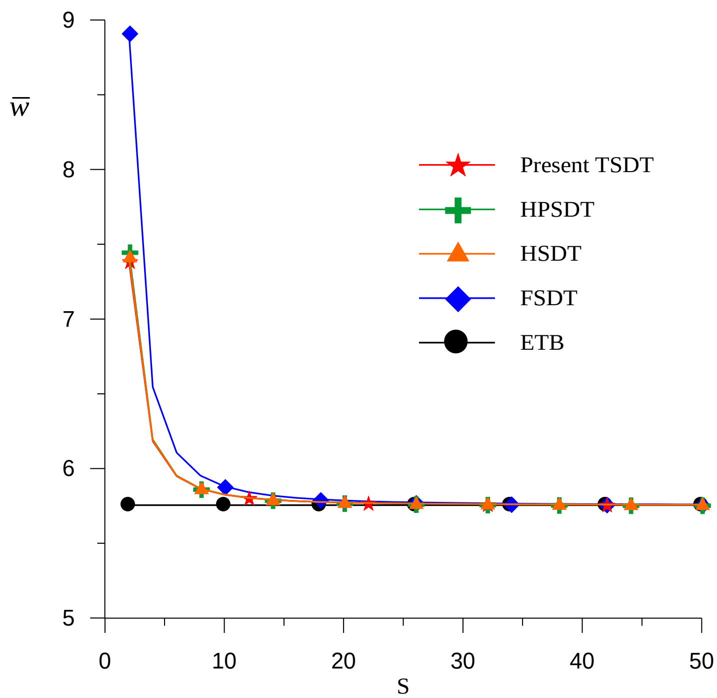

Fig. 4. Variation of maximum transverse displacement ($\bar{w}$) of beam at (x=L, z = 0) with aspect ratio S.

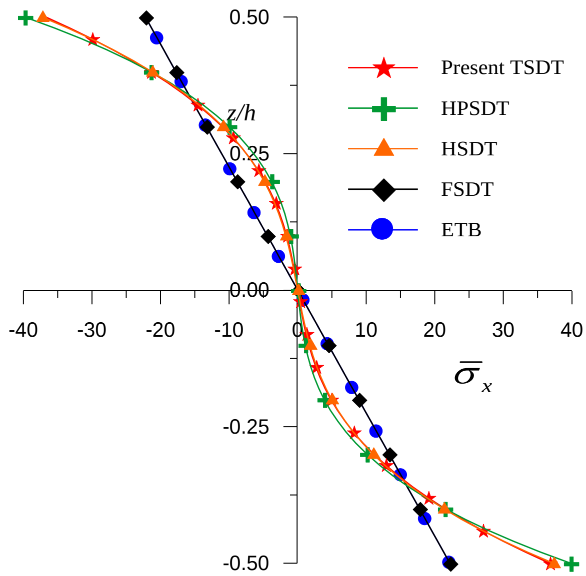

Fig. 5. Variation of axial stress ($\bar{\sigma}_x$) through the thickness of beam at (x= 0, z) for aspect ratio 4.

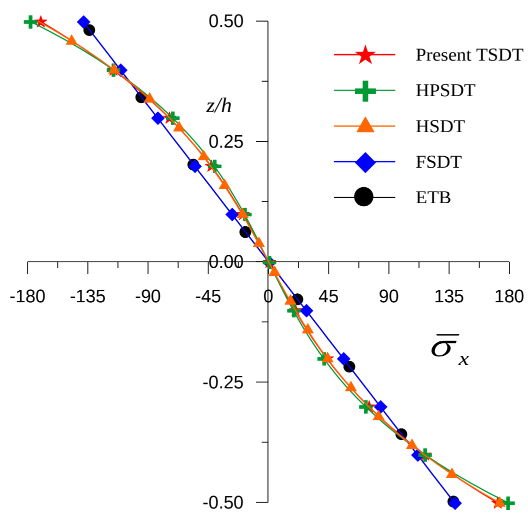

Fig. 6. Variation of axial stress ($\bar{\sigma}_x$) through the thickness of beam at (x = 0, z) for aspect ratio 10.

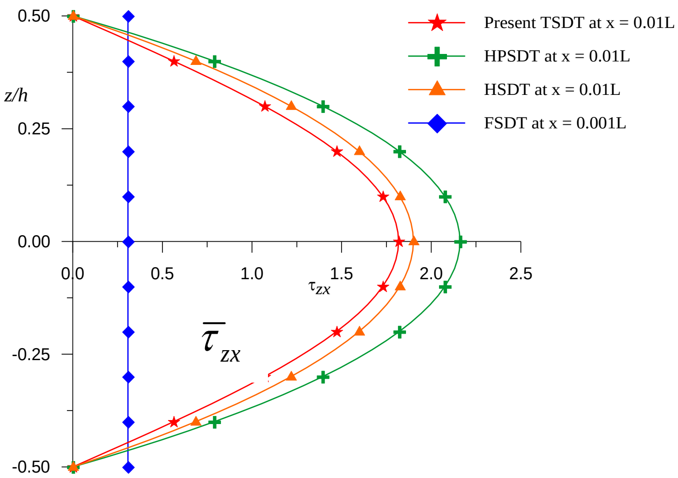

Fig. 7. Variation of transverse shear stress ($\overline{\tau}_{zx}$) through the thickness of beam at (x = 0.01L, z) obtain using CR for aspect ratio 4.

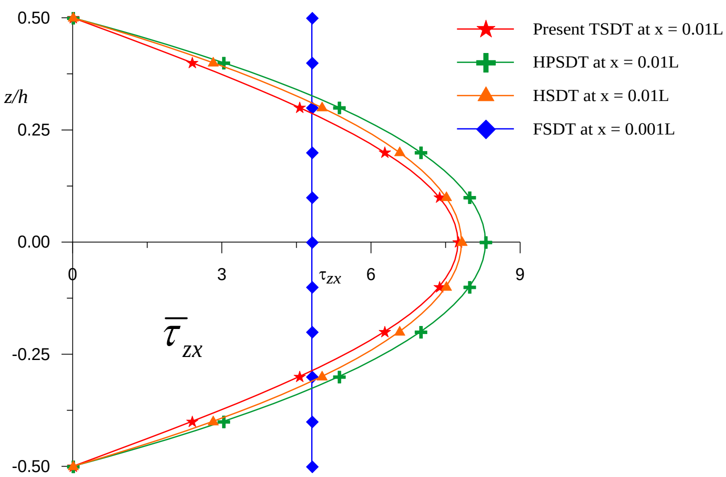

Fig. 8. Variation of transverse shear stress ($\overline{\tau}_{zx}$) through the thickness of beam at (x = 0.01L, z) obtain using CR for aspect ratio 10.

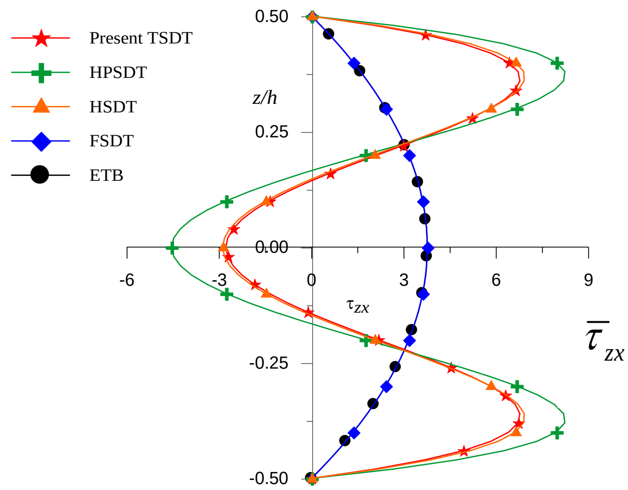

Fig. 9. Variation of transverse shear stress ($\bar{\tau}_{zx}$) through the thickness of beam at (x = 0.01L, z) obtain using EE for aspect ratio 4.

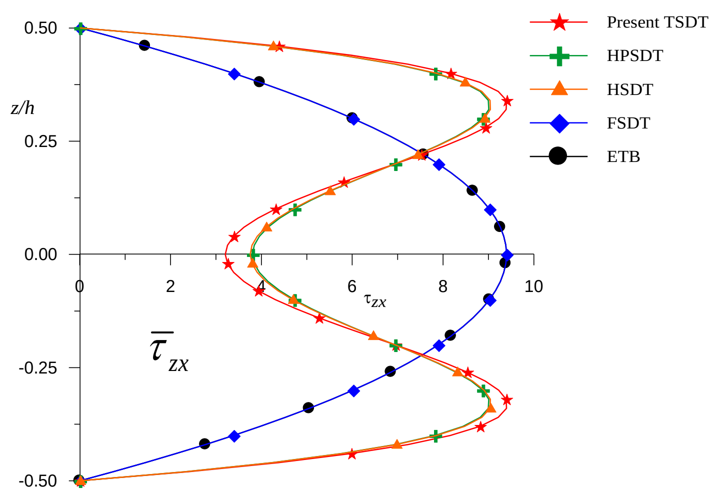

Fig. 10. Variation of transverse shear stress ($\bar{\tau}_{zx}$) through the thickness of beam at (x = 0, z) obtain using EE for aspect ratio 10.

5. Conclusions

a) The axial displacement ($\bar{u}$)

The present theory gives realistic results of this displacement component in commensurate with the other shear deformation theories. For cantilever beam with various loads, the result of present theory are nearly matching with those other higher order theory.

b) The transverse deflection ($\bar{w}$)

For cantilever beam with cosine load, the transverse deflection given by present theory is in excellent agreement with that of other higher order shear deformation theories.

c) The axial stress ($\bar{\sigma}_x$)

The axial stress and its distribution across the thickness given by present theory is in excellent agreement with that of higher order shear deformation theories.

d) The transverse shear stresses $\bar{\tau}_{zx}^{CR}$ and $\bar{\tau}_{zx}^{EE}$

For cantilever beam with cosine load, transverse shear stress and its distribution through the thickness of beam obtained from constitutive relation are in close agreement with that of other higher order refined theories; however, use of constitutive relation cannot predict the effect of stress concentration at the built-in end of the beam. The effect of stress concentration on variation of transverse shear stress is exactly predicted by the present theory with the use of equilibrium equation of two dimensional elasticity. The realistic variations of these stresses at the built-in end of various beams are presented. Hence the use of equilibrium equation is inevitable to predict the effect stress concentration in accordance with the higher / equivalent refined shear deformation theories.

In general, the use of present theory gives accurate results as seen from the numerical examples studied and it is capable of predicting the local effects in the vicinity of the built-in end of the cantilever beam. This validates the efficacy and credibility of trigonometric shear deformation theory.

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