Attenuation Models for Estimation of Vertical Peak Ground Acceleration Based on PSO Algorithm for the North of Iran

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ABSTRACT

Peak ground acceleration (PGA) is a critical parameter in ground-motion investigations, in particular in earthquake-prone areas such as Iran. In the current study, a new method based on particle swarm optimization (PSO) is developed to obtain an efficient attenuation relationship for the vertical PGA component within the northern Iranian plateau. The main purpose of this study is to propose suitable attenuation relationships for calculating the PGA for the Alborz, Tabriz and Kopet Dag faults in the vertical direction. To this aim, the available catalogs of the study area are investigated, and finally about 240 earthquake records (with a moment magnitude of 4.1 to 6.4) are chosen to develop the model. Afterward, the PSO algorithm is used to estimate model parameters, i.e., unknown coefficients of the model (attenuation relationship). Different statistical criteria showed the acceptable performance of the proposed relationships in the estimation of vertical PGA components in comparison to the previously developed relationships for the northern plateau of Iran. Developed attenuation relationships in the current study are independent of shear wave velocity. This issue is the advantage of proposed relationships for utilizing in the situations where there are not sufficient shear wave velocity data.
1. Introduction

Several major earthquakes have been occurred in Iranian Plateau that some of them are shown in Table 1. Attenuation relationships have an integral role in the analysis of seismic risk [1]. These relationships are commonly used to estimate the uncertainty in earthquake motion in conventional hazard analysis [2]. Major earthquakes in Kobe (1995), Bam (2003) and Chi Chi (1999) have highlighted the importance of the vertical component of the earthquake in the extent of damage to diverse building systems [3,4]. Numerous studies have been conducted on the importance of the vertical component of earthquake that are mentioned in the following.

Table 1
Some major earthquakes in Iran.

<table>
<thead>
<tr>
<th>Location (Province)</th>
<th>Year</th>
<th>Damage (death toll)</th>
<th>Local magnitude</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sarpol-e Zahab (Kermanshah)</td>
<td>2018</td>
<td>over 500 wounded</td>
<td>6.35</td>
</tr>
<tr>
<td>Ahar-Varzaghan (East Azarbaijan)</td>
<td>2012</td>
<td>306 killed &amp; 3037 wounded</td>
<td>6.35</td>
</tr>
<tr>
<td>Mormori (Ilam)</td>
<td>2014</td>
<td>over 250 wounded</td>
<td>6.17</td>
</tr>
<tr>
<td>Taze abad (Kermanshah)</td>
<td>2014</td>
<td>3 killed &amp; 243 wounded</td>
<td>6.17</td>
</tr>
<tr>
<td>Sang Chal (Mazandaran)</td>
<td>1957</td>
<td>1500 killed</td>
<td>6.74</td>
</tr>
<tr>
<td>Buin Zahra (Qazvin)</td>
<td>1962</td>
<td>over 12225 killed &amp; 2800 wounded</td>
<td>6.82</td>
</tr>
<tr>
<td>Dasht-e Bayaz (Southern Khorasan)</td>
<td>1968</td>
<td>about 10000 killed</td>
<td>6.8</td>
</tr>
<tr>
<td>Korizan Khaf (Khorasan)</td>
<td>1979</td>
<td>420 killed</td>
<td>6.19</td>
</tr>
<tr>
<td>North east of Kish island (Hormozgan)</td>
<td>2022</td>
<td>1 killed &amp; 30 wounded</td>
<td>5.7</td>
</tr>
<tr>
<td>Bakhtar Sarjangal (Sistan and Baluchestan)</td>
<td>2022</td>
<td>0</td>
<td>5.6</td>
</tr>
</tbody>
</table>

Terzi and Athanatopoulou (2021), by studying a historical building in Greece, concluded that the role of the vertical component on the total number of effective building cracks is significant [5]. Rahai and Arezoumandi (2008) analyzed a reinforced concrete bridge pier and they showed that vertical movements during the earthquake increased the response level and extent of the damage to the bridge [6]. They also found that vertical movements result in axial forces acting on the bridge columns which in turn, cause instability of hysteresis loops and increase the extent of damage. Consequently, in addition to the horizontal component of earthquake, the effects of the vertical component of the earthquakes should also be considered in the seismic design of structures. Vertical accelerations can change gravity loads that control the in-plane lateral load capacity of masonry foundations and affect the out of plane overturning stability of thin-walled panels [7]. Noteworthy, higher acceptable performance levels of the steel beams (larger plastic rotations) occur when both horizontal and vertical seismic components are considered [8]. The majority of researchers believe that it is important to consider the effect of the vertical component in high-rise structures and those constructed near faults [9].

So far, many efforts have been made by researchers to analyze seismic risk by looking for a suitable attenuation relationship between effective parameters of PGA such as magnitude, distance, fault depth and geotechnical conditions. Statistical modeling such as regression analysis is commonly used to estimate the relationship between two or more variables [10]. Many scholars such as Zaré (1999), Ambraseys et al. (2005), Sedaghati and Pezeshk (2017), Soghrat and Ziyaeifar (2017) and Ghodrati Amiri et al. (2018) used a variety of regression methods for the estimation of PGA [4,11–14]. In addition, optimization techniques such as the genetic
algorithm (GA) have also been used in the last decades to solve a wide variety of optimization problems such as structural damage detection optimization [15–17]. Amit Shiuly et al. (2020) used the genetic algorithm and an artificial neural network model to predict the PGA of Himalayan region in India [18]. They used the hypocenter distance and magnitude as input parameters and PGA records at different distances as output parameters. They compared their results to the result of other attenuation relationships and concluded that GA and ANN algorithms are capable to predict PGA values.

Sobhaninejad et al. (2007) used the GA with 586 data records from Europe and the Middle East to modify the attenuation relationships proposed by Ambraseys et al. (2005) [12,19]. Also, Bagheri et al. (2011) used the GA in Alborz, Central Iran and Zagros then proposed new relationships to estimate PGA and peak spectral acceleration (PSA) [15]. Regarding the lack of attenuation relationships in the vertical direction in the northern Iran plateau, the aim of this study is to extract attenuation relationships in the vertical direction using the particle swarm optimization (PSO) algorithm. To this aim, firstly, about 270 earthquake records are used to extract the initial model (an explicit equation between PGA and involved parameters), and then the PSO algorithm is applied to find the proper coefficients to predict the adequate PGA values for northern part of Iran. Finally, the results of developed attenuation relationships in this paper are compared to previously developed ones.

2. Research significance

In the present research it is tried to develop attenuation models for estimation of vertical PGA based on the PSO algorithm for the North of Iran. The importance of using such attenuation relationships, is in the seismic design of more resistant buildings and preventing more damage to buildings.

3. Methods

3.1. Particle swarm optimization (PSO) algorithm

The PSO algorithm was proposed by Eberhart and Kennedy (1995), which was inspired from the food searching behavior of birds (called particles) swarms [20]. The PSO algorithm is a subset of the swarm-based algorithms, and swarm-based algorithms are a subclass of nature-inspired algorithms, and nature-inspired algorithms are a subset of meta-heuristic algorithms. Meta-heuristic algorithms use intelligent strategies to explore a wide search space efficiently. These algorithms, in contrast to deterministic algorithms, e.g., the simplex method, because of their intelligent searching mechanisms are capable of finding global optimum effectively. In the last decades, swarm-based algorithms, especially PSO algorithm, have been widely used in different engineering problems [21–24]. The PSO algorithm in comparison to other well-known algorithms such as the genetic algorithm (GA) is simpler and easier to apply. Different comparative studies showed that the PSO algorithm was superior to other meta-heuristic algorithms in finding the final optimum solution/model [25,26]. In the PSO algorithm, individuals (birds or particles) in an n-dimensional search space are considered potential solutions. The initial swarm/population for each variable in the determined range is randomly
generated. In an iterative procedure and by fitness function (error function) the particles are evaluated and then the best particle (a particle with the lowest value of error) is introduced (called \( g_{\text{best}} \)). In addition, each particle keeps its best position (position with the lowest value of error) (called \( p_{\text{best}} \)) that has been visited by the particle so far. In the PSO algorithm, the velocity of each particle that moves towards \( p_{\text{best}} \) and also \( g_{\text{best}} \) changes frequently. If the \( k^{th} \) particle in an n-dimensional space is considered to be \( x^k = [x_1^k, x_2^k, \ldots, x_n^k] \), the \( p_{\text{best}} \) of this particle will be presented by \( (p_{\text{best}})_k = (p_{\text{best}})_{k,1}, (p_{\text{best}})_{k,2}, \ldots, (p_{\text{best}})_{k,n} \). The best particle among all particles is represented by \( g_{\text{best}} = g_{\text{best},1}, g_{\text{best},2}, \ldots, g_{\text{best},n} \) [27]. The velocity of the \( k^{th} \) particle is shown by \( v^k = [v_1^k, v_2^k, \ldots, v_n^k] \). The velocity and position of the particles are set using the following equations:

\[
\begin{align*}
     v^k_{t+1} &= w v^k_t + c_1 rand_1 \cdot ((p_{\text{best}})_k - x^k_t) + c_2 rand_2 \cdot (g_{\text{best}} - x^k_t) \\
     x^k_{t+1} &= x^k_t + v^k_{t+1}, k = 1, 2, \ldots, \text{pop}
\end{align*}
\]

where \( \text{pop} \) and \( t \), respectively, indicate the number of particles and iterations, \( v^k_t \) and \( v^k_{t+1} \) denote the velocity of the \( k^{th} \) particle in the \( t^{th} \) and \( (t+1)^{th} \) iterations, respectively. Also, \( w \) represents the inertia weight coefficient, \( c_1 \) and \( c_2 \), respectively, show acceleration factors that pull the particle towards its \( p_{\text{best}} \) and \( g_{\text{best}} \). Moreover, \( rand_1 \) and \( rand_2 \), respectively, represent uniform random numbers in the range of \([0, 1]\) and \( x^k_t \) and \( x^k_{t+1} \) are the position of \( k^{th} \) particle in the \( t \) and \( (t+1) \) iterations, respectively [20,28]. Based on the showed pseudo-code of the PSO algorithm in Figure 1, the coding was carried out in MATLAB software R2015a.

```
Begin
1: Objective function, \( F_f(X), X=(X^1, X^2, X^3, \ldots X^k)^T \)
2: Initialize each particle's velocity and position randomly, \( X^k \ (k=1, 2, 3, \ldots, \text{pop}) \)
3: Find the \( p_{\text{best}} \) and \( g_{\text{best}} \) by objective/error function
4: while \( t < \text{Max}_{\text{iteration}} \) do (termination condition)
5:     for \( k=1: \text{pop} \)
6:         Update particle velocity and particle position by Eqs.1 and 2
7:     Evaluate new solution/location
8:     Check boundary conditions and evaluate new solution/position
9:     Update the \( p_{\text{best}} \) and \( g_{\text{best}} \)
10: end for
11: end while
12: Output \( X^* \) (final solution/model)
End
```

**Fig. 1.** Pseudo-code of the PSO algorithm.

3.2. Collected data

The selected area in this study was the North Iranian Plateau, including Alborz, Azerbaijan and Kopet Dag regions, which are parts of the active tectonic regions of Iran (Figure 2a). The accelerogram data in the present study were selected from the Building and Housing Research Center (BHRC) of Iran, according to the classification of the Standard 2800 (2014), which is similar to Eurocode 8 (CEN, 2003) [29,30]. Earthquake records were classified based on the shear wave velocity of the soil at a depth of 30 m (\( V_{s30} \)), i.e., I: \( V_{s30} > 750 \); II: \( 375 \leq V_{s30} \leq 750 \); III:
175 ≤ V_{s30} ≤ 375; IV: V_{s30} < 175. Finally, about 240 records with a moment magnitude of 4.1 to 6.4 and epicentral distances less than 400 km (7 to 359 km) were selected for further analysis.

Figures 2b and 2c represent the distribution of accelerogram stations and coordinates of the event epicenter in the study area. Due to availability of the epicentral distance for all selected data in the national accelerogram network, it is used as the distance parameter. About 56.66% of whole data have the magnitude of 6-6.4, 29.16% of whole data have the magnitude of 4-5, and remained data have the magnitude of 5-6. Noteworthy, in the current study, the proposed catalog of Karimiparidari et al. (2014) was used to convert the magnitude data which were not on the moment scale [31].

![Map of the study area](image)

(a) The five active tectonic regions (Mirzaei et al. 1998) [32]

![Distribution of accelerogram stations](image)

(b) distribution of accelerogram stations

![Earthquake epicenter coordinates](image)

(c) The epicenter coordinates of the selected events

**Fig. 2.** Location of the studied region, distribution of accelerogram stations and the epicenter coordinates of events.
3.3. Error evaluation criteria

To explore the more efficient models for attenuation relationships, different evaluation criteria such as root mean squared error (RMSE), mean absolute percentage error (MAPE), mean error (ME), coefficient of determination ($R^2$) and adjusted $R^2$ ($R^2_{\text{Adjusted}}$), are used to assess the results. The RMSE and its combination with MAPE are used as the PSO objective/error function ($F_f$) for developing attenuation relationships.

Furthermore, the Log-likelihood (LLH) statistical test is used for consistency check between the measured and calculated data [33,34]. The LLH method, in order to compare two models, has widely been used in engineering sciences in the last few years. The value of the LLH criterion changes between 0 and 1, which values toward 1 show the high consistency and vice versa.

3.4. Formulation of optimization objective function

The considered objective functions in the current work, are presented by Eqs. 3 and 4. It should be noted that RMSE minimization was first considered as the objective function (Eq. 3). Regarding the results, the objective function in Eq. 4 was used to improve the optimization results where $\alpha$ and $\beta$ are factors determined by sensitivity analysis. Also, the PGA prediction equations are proposed after sensitivity analysis of several different functions based on the least squared error rate.

Objective Function: Minimize ($F_f$) = RMSE = \[ \sqrt{\frac{\sum_{i=1}^{n}(PGA_{\text{obs}}-PGA_{\text{cal}})^2}{n}} \] (3)

Objective Function: Minimize ($F_f$) = ($\alpha \times \text{MAPE} + \beta \times \text{RMSE}$) (4)

where $PGA_{\text{cal}}$ is the peak ground acceleration calculated from the attenuation relationships in Eqs. 5 and 6. Eqs. 5 and 6, respectively, estimate PGAs based on the soil and fault types (strike-slip and thrust-reverse faults).

\[
\log_{10} Y = a_1 + a_2 \exp(a_3 M_w) + a_4 \exp(a_5 R) + a_6 \exp(a_7 F_{TR}) + a_8 \exp(a_9 F_{SS}) + a_{11} \left( \frac{\sigma_{V_{S30}}}{\mu_{V_{S30}}} \right) a_{10} \]

(5)

\[
\log_{10} Y = a_1 + a_2 \exp(a_3 M_w) + a_4 \exp(a_5 R) + a_6 \exp(a_7 V_{S30}) \]

(6)

where $a_1$ to $a_{11}$ are decision variables obtained from the PSO model, $Y$ stands for PGA, $M_w$ is the moment magnitude, $R$ represents the epicentral distance (km), $V_{S30}$ shows the shear wave velocity for the top 30 m of soil (m/s), $F_{SS}$ is the strike-slip fault parameter; $F_{TR}$ is the thrust-reverse fault parameter and $\sigma_{V_{S30}}$ and $\mu_{V_{S30}}$, respectively, show the standard deviation and mean of shear wave velocities of different soil groups at a depth of 30 m.

4. Discussion

Sensitivity analysis was performed to determine the PSO parameters. According to the results, the internal parameters of the PSO algorithm, i.e., population size, maximum iteration, acceleration coefficients ($c_1$ and $c_2$) and inertia weight coefficient ($w$) were set equal to 300, 1000, 2 and 1, respectively. In this article, different forms of objective functions and attenuation relationships were tested. Finally, the best results were selected for presentation. The objective
function in Eq. 3 was used to obtain attenuation relationships for PGA, estimation. The results include the best obtained values and the standard deviation of residuals (Table 2). As seen in Table 2, the accuracy of the attenuation relationships is improved by classifying the soil based on the shear wave velocity. Table 3 presents the values obtained for the decision variables, i.e., the coefficients of attenuation relationships for different soil types with different shear wave velocities.

**Table 2**
The optimization results for estimating PGA, in the case of using RMSE as the objective function (Eq. 3)

<table>
<thead>
<tr>
<th>Soil groups</th>
<th>The best value obtained for objective function</th>
<th>Performance measures</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.114</td>
<td><strong>RMSE_T</strong> 0.292</td>
</tr>
<tr>
<td>II</td>
<td>0.327</td>
<td>MAPE_T 79.842%</td>
</tr>
<tr>
<td>III</td>
<td>0.716</td>
<td>σ_residual 0.251</td>
</tr>
<tr>
<td>All input data</td>
<td>0.292</td>
<td>RMSE_T 0.292</td>
</tr>
</tbody>
</table>

*RMSE_T is the weighted mean of RMSEs obtained for every group
**MAPE_T is the weighted mean of MAPEs obtained for every group
***σ_residual is the standard deviation of residuals

**Table 3**
The optimum values of attenuation relation coefficients for estimating PGA, in the case of using RMSE as the objective function

<table>
<thead>
<tr>
<th>Values of decision variables (coefficients of the attenuation relations)</th>
<th>a_1</th>
<th>a_2</th>
<th>a_3</th>
<th>a_4</th>
<th>a_5</th>
<th>a_6</th>
<th>a_7</th>
<th>a_8</th>
<th>a_9</th>
<th>a_{10}</th>
<th>a_{11}</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil groups</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>I</td>
<td>0.7438</td>
<td>-7.5173</td>
<td>-0.1525</td>
<td>3.1234</td>
<td>-0.0054</td>
<td>-0.2762</td>
<td>-0.0812</td>
<td>-0.0907</td>
<td>-1.9434</td>
<td>1.6421</td>
<td>-0.9170</td>
</tr>
<tr>
<td>II</td>
<td>-0.5187</td>
<td>-9.0524</td>
<td>-0.333</td>
<td>3.0502</td>
<td>-0.0152</td>
<td>0.2737</td>
<td>-1.7204</td>
<td>0.7468</td>
<td>-4.5034</td>
<td>0.8724</td>
<td>-2.9745</td>
</tr>
<tr>
<td>III</td>
<td>3.2382</td>
<td>-6.6793</td>
<td>-0.2334</td>
<td>1.2775</td>
<td>-0.0118</td>
<td>-3.1572</td>
<td>-3.8436</td>
<td>-2.9659</td>
<td>-5.0928</td>
<td>1.7035</td>
<td>2.4601</td>
</tr>
<tr>
<td>All input data</td>
<td>-0.8350</td>
<td>-10.00</td>
<td>-0.4256</td>
<td>2.0401</td>
<td>-0.0137</td>
<td>7.0245</td>
<td>-4.3108</td>
<td>0.2116</td>
<td>0.5903</td>
<td>-0.3303</td>
<td>-4.1501</td>
</tr>
</tbody>
</table>

The general objective function, $\alpha \times MAPE + \beta \times RMSE$ (Eq. 4), was used to improve the accuracy of attenuation relationships for PGA, estimation. After sensitivity analysis based on the different values of coefficients $\alpha$ and $\beta$, the objective functions MAPE+2RMSE and MAPE+3RMSE were, respectively, selected for the Tabriz-Kopeh Dagh and Alborz-Khazar faults. In Table 4 acceptable results are shown. Comparing the results in Table 2 and 4 indicates an improvement of MAPE using the combined objective function. Table 5 lists the values that obtained for the coefficients of the attenuation relationship for different fault types.

**Table 4**
The optimization results for estimating PGA, in the case of using RMSE and MAPE as the hybrid objective function (Eq. 4)

<table>
<thead>
<tr>
<th>Fault types</th>
<th>The best value obtained for objective function</th>
<th>Performance measures</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MAPE</td>
</tr>
<tr>
<td>Tabriz-Kopeh Dagh</td>
<td>1.003</td>
<td>39.821%</td>
</tr>
<tr>
<td>Alborz-Khazar</td>
<td>1.718</td>
<td>46.422%</td>
</tr>
</tbody>
</table>

*RMSE_T is the weighted mean of RMSEs obtained for each group
**MAPE_T is the weighted mean of MAPEs obtained for each group
***σ_residual is the standard deviation of residual
Table 5
The optimum values of attenuation relation coefficients for estimating PGA, in the case of using RMSE and MAPE as the hybrid objective function.

<table>
<thead>
<tr>
<th>Fault Types</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>$a_3$</th>
<th>$a_4$</th>
<th>$a_5$</th>
<th>$a_6$</th>
<th>$a_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tabriz- Kopeh Dagh</td>
<td>0.19602</td>
<td>-8.7819</td>
<td>-0.2684</td>
<td>2.1508</td>
<td>-0.01614</td>
<td>-4.0111</td>
<td>-9.5617</td>
</tr>
<tr>
<td>Alborz- Khazar</td>
<td>-0.7448</td>
<td>9.9998-</td>
<td>-0.4495</td>
<td>1.4815</td>
<td>-0.01645</td>
<td>2.4233</td>
<td>-3.5297</td>
</tr>
</tbody>
</table>

4.1. Comparing the proposed attenuation relationships with others

Derived attenuation relationships in this study were compared with those presented in the literature for the north plateau of Iran. To this aim, five attenuation relationships (Table 6) that have been proposed for Europe-Middle East and Iran (North Iranian Plateau) were selected to compare with the proposed relationship in this study.

Table 6
Candidate attenuation relationships.

<table>
<thead>
<tr>
<th>Attenuation Models</th>
<th>Abbreviation</th>
<th>Type of Distance</th>
<th>$M_a$ Range</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nowroozi (2005) [35]</td>
<td>N-5</td>
<td>Epicentral</td>
<td>5.1-6.4</td>
</tr>
<tr>
<td>Ghodrati Amiri et al. (2007) [36]</td>
<td>G-7</td>
<td>Focal</td>
<td>5.1-6.4</td>
</tr>
<tr>
<td>Sedaghati and Pezeshk (2017) [13]</td>
<td>SP-17</td>
<td>Boore-Joyner</td>
<td>3.2-6.4</td>
</tr>
</tbody>
</table>

The evaluation indices including $R^2$, $R^2_{\text{Adjusted}}$, ME, RMSE, MAPE and LLH were calculated for different attenuation relationships and the results were presented in Table 7. In this table, PGA$_{\text{a}-1}$ is the proposed model based on the shear wave velocity and objective function of RMSE for PGA, and PGA$_{\text{a}-3}$ is the proposed model based on the fault type and hybrid objective function of MAPE and RMSE for PGA. According to the results, PGA$_{\text{a}-1}$ has a lower RMSE than the other selected relationships. The PGA$_{\text{a}-1}$ and PGA$_{\text{a}-3}$ models with a higher $R^2$ and $R^2_{\text{Adjusted}}$ outperformed than other relationships. The PGA$_{\text{a}-3}$ and SZ-17 models showed lower MAPE values than other ones. However, PGA$_{\text{a}-3}$ outperformed SZ-17 in terms of other evaluation criteria (Table 7). Based on ME, the PGA$_{\text{a}-3}$ and PGA$_{\text{a}-1}$ predicted PGA values lower and higher than their recorded values, respectively. Moreover, PGA$_{\text{a}-3}$ with a lower LLH outperformed the selected attenuation relationships. The lowest LLH was obtained for PGA$_{\text{a}-1}$. The attenuation relationships are ranked in Table 8 based on different assessment criteria. According to Table 8 and all assessment criteria, it could be concluded that the proposed attenuation relationships in this article outperform other attenuation relationships.

Table 7
The values of error evaluation criteria for PGA, estimation relations

<table>
<thead>
<tr>
<th>Models</th>
<th>MAPE%</th>
<th>RMSE</th>
<th>ME</th>
<th>$R^2$</th>
<th>$R^2_{\text{Adjusted}}$</th>
<th>LLH</th>
</tr>
</thead>
<tbody>
<tr>
<td>*PGA$_{\text{a}-1}$(Present Study)</td>
<td>79.843</td>
<td>0.22</td>
<td>0.012</td>
<td>0.593</td>
<td>0.586</td>
<td>0.049</td>
</tr>
<tr>
<td>**PGA$_{\text{a}-3}$(Present Study)</td>
<td>42.682</td>
<td>0.316</td>
<td>0.081</td>
<td>0.999</td>
<td>0.991</td>
<td>0.386</td>
</tr>
<tr>
<td>SZ-17</td>
<td>41.623</td>
<td>0.365</td>
<td>0.073</td>
<td>0.152</td>
<td>0.136</td>
<td>0.612</td>
</tr>
<tr>
<td>SP-17</td>
<td>50.670</td>
<td>0.425</td>
<td>0.117</td>
<td>0.039</td>
<td>0.013</td>
<td>0.815</td>
</tr>
<tr>
<td>N-5</td>
<td>55.411</td>
<td>0.342</td>
<td>0.069</td>
<td>0.209</td>
<td>0.202</td>
<td>0.500</td>
</tr>
<tr>
<td>Z-9</td>
<td>66.267</td>
<td>0.376</td>
<td>0.108</td>
<td>-0.024</td>
<td>-0.043</td>
<td>0.740</td>
</tr>
<tr>
<td>G-7</td>
<td>103.569</td>
<td>0.639</td>
<td>0.078</td>
<td>0.179</td>
<td>0.138</td>
<td>1.400</td>
</tr>
</tbody>
</table>

*The proposed model based on the shear wave velocity and objective function of RMSE for PGA, **The proposed model based on the fault type and hybrid objective function of MAPE and RMSE for PGA,
Table 8
Ranking of PGA<sub>v</sub> estimation models.

<table>
<thead>
<tr>
<th>Models</th>
<th>MAPE%</th>
<th>RMSE</th>
<th>ME</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;</th>
<th>R&lt;sup&gt;2&lt;/sup&gt;Adjusted</th>
<th>LLH</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA&lt;sub&gt;v&lt;/sub&gt;-1(Present Study)</td>
<td>6</td>
<td>1</td>
<td>1</td>
<td>2</td>
<td>2</td>
<td>1</td>
</tr>
<tr>
<td><strong>PGA&lt;sub&gt;v&lt;/sub&gt;-3(Present Study)</strong></td>
<td>2</td>
<td>2</td>
<td>5</td>
<td>1</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>SZ-17</td>
<td>1</td>
<td>4</td>
<td>3</td>
<td>5</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>SP-17</td>
<td>3</td>
<td>6</td>
<td>7</td>
<td>6</td>
<td>6</td>
<td>6</td>
</tr>
<tr>
<td>N-5</td>
<td>4</td>
<td>3</td>
<td>2</td>
<td>3</td>
<td>3</td>
<td>3</td>
</tr>
<tr>
<td>Z-9</td>
<td>5</td>
<td>5</td>
<td>6</td>
<td>7</td>
<td>7</td>
<td>5</td>
</tr>
<tr>
<td>G-7</td>
<td>7</td>
<td>7</td>
<td>4</td>
<td>4</td>
<td>4</td>
<td>7</td>
</tr>
</tbody>
</table>

*The proposed model based on the shear wave velocity and objective function of RMSE for PGA<sub>v</sub>*

**The proposed model based on the fault type and hybrid objective function of MAPE and RMSE for PGA<sub>v</sub>*

For further investigation and a better comparison of the proposed attenuation relationships with other attenuation relationships, the observed and estimated PGA<sub>v</sub> values are presented in Figure 3. As seen, there is greater compliance between the accelerogram and calculated data for the PGA<sub>v</sub>-3 model.

For further assessment of the attenuation models to each other, they were investigated and compared in terms of accuracy and standard deviation of residuals in relation to the moment magnitude (M<sub>W</sub>). The results are presented in Table 9 where P<sub>a</sub> and P<sub>b</sub>, respectively, represent the p-values of the slope and y-intercept. The null hypothesis is rejected when the p-values are lower than or equal to the significance level of 5%. A p-value greater than 5% leads to confirmation of the null hypothesis. When the p-values approach 1, the resulting attenuation model with a lower standard deviation will be capable to predict PGA with the higher accuracy.

Table 9
Deviation of residuals versus moment magnitude for PGA<sub>v</sub> estimation models.

<table>
<thead>
<tr>
<th>Models</th>
<th>(P&lt;sub&gt;a&lt;/sub&gt;)&lt;sub&gt;Mw vs residual&lt;/sub&gt;</th>
<th>(P&lt;sub&gt;b&lt;/sub&gt;)&lt;sub&gt;Mw vs residual&lt;/sub&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>PGA&lt;sub&gt;v&lt;/sub&gt;-1(Present Study)</td>
<td>0.354</td>
<td>0.410</td>
</tr>
<tr>
<td>PGA&lt;sub&gt;v&lt;/sub&gt;-3(Present Study)</td>
<td>0.743</td>
<td>0.385</td>
</tr>
<tr>
<td>SZ-17</td>
<td>0.575</td>
<td>0.957</td>
</tr>
<tr>
<td>SP-17</td>
<td>0.519</td>
<td>0.244</td>
</tr>
<tr>
<td>N-5</td>
<td>0.162</td>
<td>0.068</td>
</tr>
<tr>
<td>Z-9</td>
<td>0.894</td>
<td>0.422</td>
</tr>
<tr>
<td>G-7</td>
<td>0.258</td>
<td>0.224</td>
</tr>
</tbody>
</table>

*The P-value of fitted line slope on the residuals (distribution of models residuals versus M<sub>w</sub>)*

**The P-value of fitted line intercept on the residuals (distribution of models residuals versus M<sub>w</sub>)**
Fig. 3. The comparison of the attenuation relationships for estimating PGAv.
According to Table 9, Z-9 followed by PGA_v-3 has the lowest standard deviation due to a high P-value. However, as shown in Table 7, PGA_v-3 has a significantly lower ME than Z-9 and thus is superior to Z-9. Figure 4 shows the line fitted on the residuals versus the moment magnitude for the best and worst PGA_v estimation attenuation models.

![Residual distribution versus M_w for the best and the worst PGA_v estimation relations](image)

**Fig. 4.** Residual distribution versus M_w for the best and the worst PGA_v estimation relations.

Considering MW=6 and Vs30=500 m/s, the PGA_v was estimated for different epicentral distances using different attenuation relationships (Figure 5). The PGAv-3 output has the maximum consistency with the SZ-17 and N-5 output in the vertical direction. There is a small gap between the SP-17 diagram and other mentioned diagrams (the type of distance in this relation is different from the other relations). The G-7 due to the lack of accelerograph data is very different from other charts.

![A comparison between the attenuation models for the horizontal and vertical directions considering M_w=6, Vs30=500 m/s (SC=II) for vertical direction.](image)

**Fig. 5.** A comparison between the attenuation models for the horizontal and vertical directions considering M_w=6, Vs30=500 m/s (SC=II) for vertical direction.

5. Conclusion

In this study, regarding the importance of the vertical component of PGA in the extent of damage to buildings, PGA estimation in the vertical direction was considered. About 240 earthquake data were gathered from the BHRC with a moment magnitude of 4.1 to 6.4 and the epicentral distance of less than 400 km. Then, using the PSO algorithm, new attenuation relationships were developed for seismic regions in the northern plateau area of Iran. Using different evaluation criteria and the LLH test, the proposed attenuation relationships were compared with those
presented in the literature for the north plateau of Iran, Europe and the Middle East. The obtained results are summarized in following:

- PSO algorithm can be used as a powerful optimization method to improve the accuracy of attenuation relationships.
- The hybrid objective function considered in this study is very effective in improving the accuracy and efficiency of the proposed attenuation models.
- In contrast to the previous attenuation relationships presented by other researchers, some attenuation relationships proposed in this study do not need the shear wave velocity data for the estimation of ground motion acceleration. This characteristic facilitates their application in regions where the shear wave velocity data is not available.
- The attenuation models proposed in this paper outperformed other attenuation relationships in terms of different evaluation criteria and they were relatively highly accurate in the estimation of the PGA.
- The model proposed for estimating PGA, with the combined objective functions and based on the classification of fault types was highly consistent with the model presented by Soghrat and Ziyaifar (2017).

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**Conflicts of interest**

The authors declare no conflict of interest.

**References**


