Journal of Soft Computing in Civil Engineering 4-4 (2020) 47-60

Contents lists available at SCCE

Journal of Soft Computing in Civil Engineering

Journal homepage: www.jsoftcivil.com

Assessment of the Modulus of Elasticity at a Triaxial Stress State for Rocks Using Gene Expression Programming

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https://doi.org/10.22115/SCCE.2020.232778.1223

ARTICLE INFO

Article history:
Received: 27 May 2020
Revised: 21 September 2020
Accepted: 23 September 2020

Keywords:
Triaxial stress state;
Confining stress;
Modulus of elasticity;
Modeling;
Gene expression programming.

ABSTRACT

Rocks were subjected to the deformation test under five different confining stresses (0, 5, 10, 15, and 20 MPa) using the Hoek cell to determine changes in the elastic properties of the rocks under confining stress, and the results were evaluated based on density, porosity, Schmidt hardness, and compressive strength. A total of nine different rocks, two granites, two andesites, two limestones, one tuff, one diorite, and one marble, were used. When the confining stress was increased from 0 MPa to 5 MPa and from 5 MPa to 10 MPa, elasticity increased by approximately 20%. When the confining stress was increased from 10 MPa to 20 MPa, the increase was 7% in comparison with the previous value. Then, to formulate the modulus of elasticity for rocks under the triaxial stress conditions, a new and intelligent approach to gene application, gene expression programming was utilized. The success of the model was thoroughly assessed based on measurable criteria such as the root mean square error, mean absolute percentage error, and coefficient of determination. Furthermore, the success of the model was comprehensively assessed based on the model testing, and 0.88 and 0.81 R^2 values were obtained for training and validation, respectively. The performance of the gene expression programming-based formulation was compared with the formulae previously proposed in the literature. The gene expression method exhibited the best performance, and it was identified to calculate the modulus of elasticity under triaxial stress conditions more effectively.

How to cite this article: Atici U. Assessment of the modulus of elasticity at a triaxial stress state for rocks using gene expression programming. J Soft Comput Civ Eng 2020;4(4):47–60. https://doi.org/10.22115/scce.2020.232778.1223.

1. Introduction

The characterization of the engineering properties of rocks plays a vital role in planning the engineering structure in terms of stability and cost analysis. The modulus of elasticity (E) represents an essential parameter for understanding the deformation behavior of rocks and is among the most significant mechanical properties of rocks concerning their areas of usage. The said parameter is crucial in tunnel projects, slope consistency, pillar configuration, rock destruction and drilling, dams, and numerous other civil and mining applications. It has been widely utilized to analyze structural deformations, shrinkage, creep, crack control, etc.

The modulus of elasticity is used for the purpose of determining the deformation properties of wood, metal, concrete, rocks, and many other materials. The mentioned parameter represents the ratio of stress to corresponding strain in material under tension or compression. It may be obtained from unconfined compressive strength (G_1) tests and the incline of tensile test stress-strain diagrams for rocks. The deformation test performed to determine the modulus of elasticity is difficult, time-consuming, and requires particular specimens and equipment. Many authors have tried to identify this parameter using more easily identified material properties due to the difficulties in determining the modulus of elasticity using regression analysis [1–4], genetic programming applications, e.g. artificial neural networks [4–8], fuzzy logic [9–11], and genetic algorithms [12–14].

The elasticity test is performed only under uniaxial stresses. Nevertheless, in practice, underground rocks are also affected by confining stress (G_3), assuming that G_2= $G_3 \neq 0$ and the modulus of elasticity (E) may be insufficient to reflect the actual conditions of rocks. In real situations, rocks are exposed to confining stress, and the effects of their elasticity characteristics under confining stress vary. In previous studies, it was stated that the strength and the modulus of elasticity of rocks would increase with the increase in G_3. However, there are not many studies in the literature on the rate of this increase and estimating the respective values. Liang et al. [15] and Ma et al. [16] investigated the stress-strain relationships for rock salt and anhydrite. On the contrary, Li et al. [17] reported that the modulus of elasticity was independent of strain rate. Hoek and Brown [18], Arora [19], and Asef and Reddish [20] developed a different formula for predicting the modulus of elasticity with confining pressure.

This study's goal was to predict E at a triaxial stress state using a novel approach, gene expression programming (GEP). The complicated relationship between the parameters affecting the E value may be easily modeled using the GEP approach, unlike classical methods. Two hundred fifty-six data sets were collected from experimental studies using nine different rocks (two granites, two andesites, two limestones, one tuff, one diorite, and one marble). E was used as an output, and five parameters, including G_3 (MPa), G_1 (MPa), porosity (g/cm^3) (θ), ultrasonic sound velocity (km/s), and Schmidt rebound hardness (SH), were used as the input parameters. The model was trained with 192 data sets of the experimental results, following which 64 data sets were utilized for testing. The result of the obtained GEP-based formulation was tested and compared to the formulations proposed by Arora [19] and Asef and Reddish [20] using experimental data. It was observed that the findings acquired using the model corresponded to the experimental data at a high rate.

2. Material and experimental methods

2.1. Rock characteristics

The physical characteristics of the sampled rocks were identified by θ and ultrasonic P-wave velocity. The samples were cored from large blocks in the laboratory, and their testing was carried out following the procedures described in ISRM [21] . Density (ρ) and θ were found by employing the water saturation and caliper techniques on the NX-sized samples, using five samples every time. The high-frequency ultrasonic pulse technique described by ASTM [22] was employed to measure the velocities of the P (compression) waves using the 1 MHz nominal frequency. The wave velocity through the sample was computed using the travel time from the transmitter to a receiver. A minimum of five measurements was made from every rock type, and they were averaged.

The measured mechanical properties were UCS and rebound tests, which were determined in accordance with the standards by ISRM [21]. A minimum of five core samples from every rock were exposed to strength tests, which were conducted using a fully automatic, servo-controlled, instrumented, and computer-controlled press machine. Rebound tests were conducted on the block samples using the Digi-Schmidt 2000 concrete test hammer. At least twenty measurements at various points of every sample were performed for the rebound hardness of rocks. Table 1 summarizes the physical and mechanical characteristics of the rocks examined. This table shows that the standard deviation values were high because of working with natural materials.

Table 1
Basic physical and mechanical characteristics of rocks.

	Min.	Max.	Mean	Std. Dev.
θ (%)	0.36	18.23	4.36	6.16
ρ (gr/cm^3)	1.85	2.86	2.57	0.33
Vp (Km/s)	2.17	5.82	4.28	1.38
SH	43.00	70.60	58.18	8.05
G_1 (MPa)	28.00	167.97	81.14	34.95
E (MPa)	1278.20	28260.20	10854.07	5513.98

2.2. Elasticity at a triaxial stress state

Triaxial compression tests with different G_3 were carried out according to ISRM [21] using a hydraulic servo-control test system comprised of three major sub-systems, including a servo-controlled press machine, automatic axially loading unit, instrumentation, and a personal computer. G_3 was achieved through hydrostatic pressure using an automatic pressure system for confining stress in a Hoek triaxial cell. A series of G_3 at 5, 10, and 15 MPa and at 20 MPa for only two rocks (one limestone and one granite) were applied as confining pressure. The maximum confining stress was limited to 20 MPa, which was equivalent to a load of approximately 800 m depth representing the limit of the depth of most engineering works. Axial loading and G_3 were simultaneously increased to the pre-established values, following which G_3 was fixed, and axial loading continued to be increased at a constant speed up to the pre-determined stresses. The axial strains of the test samples were measured by the linear variable differential transformer (LVDT) using the software prepared for controlling the press machine. A PC continuously showed and performed the recording of the axial load and the axial displacement during the test. The tangential *E* was derived as a tangent to the stress-strain curve ($E=\Delta G/\Delta \epsilon$) at 50% of the maximum stress (G_1) derived (Fig. 1). The E values for different G_3 values of the nine various rocks used in the study are presented in Fig. 2. It is observed that when G_3 was increased from 0 to 5 and from 5 to 10, the increase in the modulus of elasticity was 20%, and when G_3 was increased from 10 to 15, the mean modulus of elasticity increased by 7% on average in the used rocks.

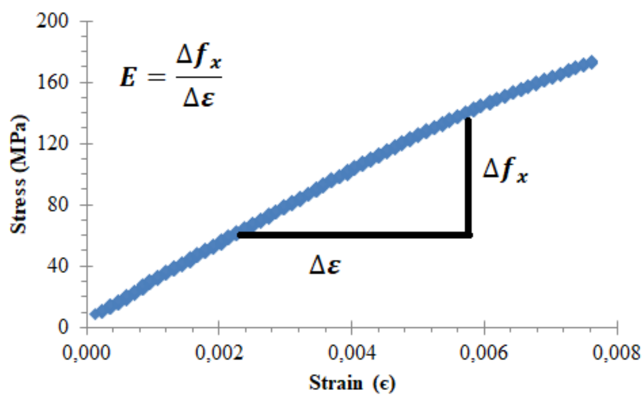


Fig. 1. Determination of the modulus of elasticity.

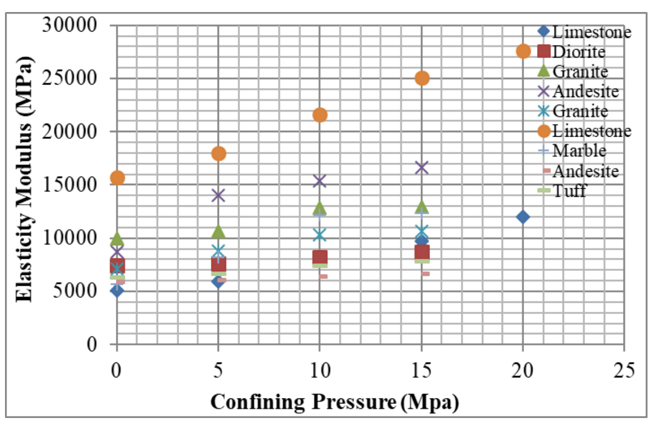


Fig. 2. The correlation between E and G_3.

The effects of G_1 and G_3 on the E values are shown in Fig. 3. In particular, when G_3 was applied as 5 and 10 MPa, it was observed that the modulus of elasticity was about 20% higher compared to the previous value. When G_3 was applied as 20 MPa, the modulus of elasticity continued to increase. However, this rate was only 7% higher than the value at 15 MPa. The most important reason for this was the fact that the discontinuities in the studied rocks were continuous to trap up to 10 MPa. After 10 MPa, the closing procedure was continued, but the effect on deformation was reduced. In case of the low confining stress, the rocks with low G_1 and high θ regarding the increase in the modulus of elasticity were relatively low. When the confining stress increased, the increase in the modulus of elasticity was higher. However, in the rocks with high G_1 and low θ, the situation was the opposite (Fig. 4). Likewise, the highest rate of increase in the modulus of elasticity was observed in rocks with the highest SH as the confining stress increased (Fig. 5).

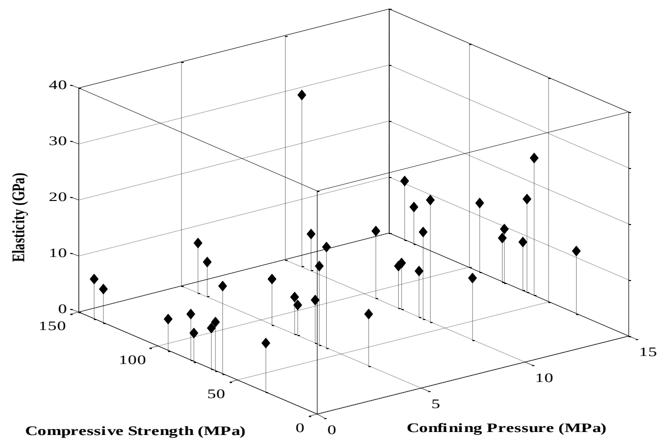


Fig. 3. The correlation between E and G_1, G_3.

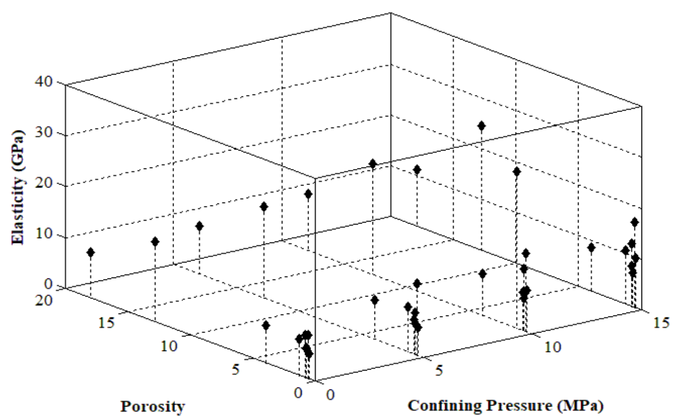


Fig. 4. The correlation between E and θ, G_3.

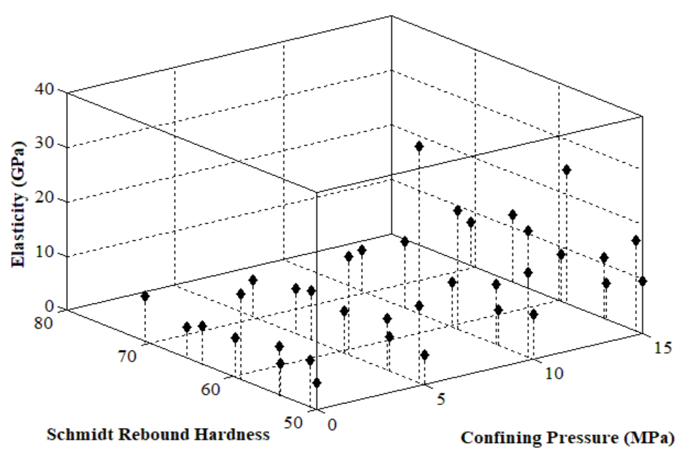


Fig. 5. The correlation between E and SH, G_3.

3. Gene expression programming (GEP)

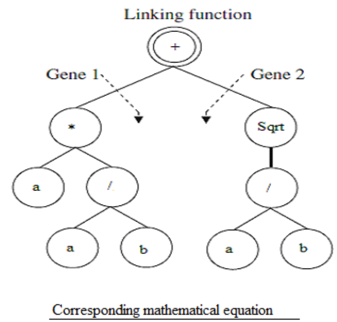
Although classical modeling methods such as regression analysis have many advantages, these methods need a correlation between independent and dependent variables [23]. The significant weakness of traditional modeling methods is their being not equipped for providing useful prediction conditions for complicated cases. The soft computing approach surpasses the problems and disadvantages of regression analysis and yields incredible and accurate results [9]. To overcome this limitation, new methodologies have been proposed. The researchers have produced rearranged prediction conditions without accepting an earlier type of the current relationship. Gene expression programming (GEP) is another amazing, transformative AI-based technique created by Ferreira [24]. It is a groundbreaking developmental calculation that combines both the basic direct chromosomes of a settled length like the ones utilized in hereditary calculations and the ramified structures of various sizes and shapes like the parse trees of hereditary programming. Its assessment arrangement of any information resembles that of the biological assessment and is a PC program that is encoded in direct chromosomes of a settled length. In the mentioned methodology, a scientific capacity characterized as a chromosome with multiple qualities is produced by utilizing the information exhibited to it.

To create a GEP demonstration, five segments, including the function set, terminal set, control parameters, fitness function, and stop condition, are needed. After encoding the issue for the competitor arrangement and determining the wellness work, the calculation randomly makes an underlying populace of reasonable people (chromosomes) and afterward, changes over every chromosome into an articulation tree related to a scientific articulation. A short time later, the anticipated target is contrasted, and the real one and the wellness score for every chromosome are resolved. If it is adequately high, the calculation stops. Nevertheless, a part of the chromosomes are chosen by utilizing roulette wheel testing, and after this process, they are transformed to acquire the new ages. This closed cycle proceeds until the wanted wellness score is achieved, following which the decoding of the chromosomes is performed for the best arrangement of the issue [25,26].

GEP has two languages, such as the language of qualities and the language of ETs. In GEP, on account of the basic decisions about the structure of ETs and their associations, it is reasonable to promptly induce the phenotype presented as the arrangement of quality and vice versa. This comprehensible bilingual documentation is named the Karva language [24]. For example, a numerical articulation $[a * (a/b)] + [\sqrt[2]{a/b}]$ may be expressed by a two-quality chromosome or an ET, as seen in Fig. 6. The said figure indicates how two qualities are encoded as a straight string and how they are communicated as an ET. The reader is referred to the study conducted by Ferreira [24,27] to obtain more detailed information.

3.1. Overview of gene expression programming

Due to the failed linear, nonlinear either single or multiple regression in terms of providing the suitable outcomes, in this study, the principal point of improvement of GEP-based definitions was to produce the numerical capacities for the prediction of E at 5, 10, 15, and 20 MPa G_3 at a triaxial stress state. While training and testing the GEP model, G_3, density (ρ), θ, SH, Vp, and G_1 were used as the input variables, whereas E was used as the output variable.



$$\left\{\left[a * \left(\frac{a}{b}\right)\right] + \left[\sqrt[2]{\frac{a}{b}}\right]\right\}$$

Fig. 6. The case of the GEP expression tree.

The GEP model was created with 256 data sets of the rock samples acquired from this experimental research. The numbers of the experimental data sets utilized for initial practices and testing/validation in the mentioned model were 191 and 63, respectively. Table 2 summarizes the parameters utilized in the development of the GEP model.

Many GEP models were tested to find the best model using the GeneXproTools software. Four basic arithmetic operations (+, −, ∗, /) and several primary mathematical functions (Exp, Ln, X^2, 3Rt, Atan, Min, Max, Avg2, Tanh, Not, Inv) were used, and the RMSE limitation was determined as the fitness function.

In the GEP analysis, the setting parameter values significantly affected the output model's fitness. They contained the chromosome number, the number of genes, the head size of the gene, and the ratio of genetic operators. To select the chromosomal tree, in other words, the head's length and the gene number, the GEP models primarily utilized a single gene and two lengths of heads and increased the number of genes and heads, consecutively, in every run, while the testing and training performance of every model was monitored. Following a number of tests, for the models, the numbers of genes 3 and eight heads were revealed to yield the best outcomes. The linking of the sub-ETs (genes) was performed as a result of subtraction.

Table 2
GEP parameters utilized for the models created.

Definition of the parameter	GEP Model
Fitness function	RMSE
Number of chromosomes	30
Gene number	3
Mutation rate	0.00138
Inversion rate	0.00546
One-point recombination rate	0.00277
Two-point recombination rate	0.00277
Gene recombination rate	0.00277
Gene transposition rate	0.00277
Literals	17
Arithmetic operators	+ - x /
Mathematical function	Exp, Ln, X^2, 3Rt, Atan, Min, Max, Avg2, Tanh, Not, Inv
Head size	8
Tail Size	9
Gene Size	26
Linking function	Subtraction

Finally, an association of all genetic operators, including transposition, mutation, and crossover, was utilized as the set of genetic operators. The chromosome 30 was determined as the best generation of individuals in estimating E. The accurate formulation after replacing the constants based on the constructed models for E is given by the following formula:

$$E = \left\{68.564\left(\frac{10^{V_p}}{Min(f_x;SH)} * (\theta * f_x\right)\right\} - \left\{Min(2094.579V_p^2;\left(\left(V_p + \rho\right) * (\sigma_3 * SH\right)\right\} - \left\{\left[\left(\frac{(SH+\sigma_1)+(V_p*\sigma_1}{2}\right) * (\rho * SH)\right] * ATan(-17.39\theta)\right\} \quad (1)$$

Fig. 7 presents the expression tree of the formulation for the model, where d0, d1, d2, and d3 denote G_3, ρ, θ, SH, Vp, and G_1. Table 3 contains the list of the constants utilized in the formulation.

Table 3
Coefficients used in the GEP model.

Constant	Gene I	Gene II	Gene III
C0	8.45	4.09	-2.93
C1	6.19	8.65	6.02
C2	-4.61	-4.79	1.65
C3	5.49	-204.14	7.46
C4	2.65	8.53	-2.04
C5	-0.39	-2.55	-9.48
C6	-6.03	-8.69	-17.39
C7	6.19	-5.13	1.67
C8	9.13	0.27	9.00
C9	-9.63	9.27	-5.84

4. Results and discussion

The current section investigates the findings of the analysis of the created model and a quantitative evaluation of the model's predictive abilities. In assessing the model, defining the model's performance and the criteria for estimation accuracy is essential. To produce the GEP models and demonstrate the prediction ability of the GEP, the database consisting of the test information was separated into two sets, training and testing. From the 256 data sets, for the training and testing processes, 192 and 64 experimental data sets, respectively, were selected in a random way.

To compare the performance of the model with the previous models in the literature, three previously utilized models were determined. These models are the models of Hoek and Brown [18], Arora[19], and Asef and Reddish [20].

Hoek and Brown developed a highly accepted formula that uses the modulus of elasticity, strain, Poisson's ratio, and confining pressure. This is shown by equation (2). However, it is both very expensive and technically difficult to determine the Poisson's ratio used in this equation under triaxial stress conditions.

Fig. 7. Expression tree of the GEP approach model.

$$E\varepsilon_1 = \sigma_1 - \vartheta(\sigma_2 + \sigma_3) \quad (2)$$

Where ϑ is the Poisson′s ratio.

Arora suggested a model using three different rock types (plaster of Paris, Jamrani sandstone, and Agra sandstone) upon the estimation of the modulus of elasticity with confining pressure. This empirical equation is given below:

$$\frac{E_{(\sigma_3=0)}}{E_{(\sigma_3=\sigma_2\neq 0)}} = 1 - exp\left(-0.1\frac{\sigma_1}{\sigma_3}\right) \quad (3)$$

Asef and Reddish developed the model with its own test data and proposed a new model. It is stated that this model illustrates a significant improvement by obtaining the coefficient of determination (R^2:0.69) with the original data of Arora and its own experimental data using arkose sandstone. This empirical equation is presented below:

$$\frac{E_{(\sigma_3=\sigma_2\neq 0)}}{E_{(\sigma_3=0)}} = \frac{200\left(\frac{\sigma_3}{\sigma_1}\right)+b}{\left(\frac{\sigma_3}{\sigma_1}\right)+b} \quad (4)$$

Where $b=15+60e^{-0.18G1}$, $E_{(G3=0)}$ is the modulus of elasticity at the unconfined compressive strength (GPa), $E_{(G3=\ G2\neq 0)}$ is the modulus of elasticity at a given confining stress (GPa), G_3 and G_1 are the confining stress (MPa) and unconfined compressive strength (MPa), respectively.

The results of the created model utilizing GEP and models proposed by Arora and Ased and Reddish were assessed based on the root mean square error (RMSE) values, mean absolute percentage error (MAPE), and the coefficient of determination (R^2). The above-mentioned criteria are presented in equations 5, 6, and 7, respectively.

$$MAPE = \frac{1}{n}\left[\frac{\sum_{i=1}^{n}|t_i - o_i|}{\sum_{i=1}^{n} t_i} * 100\right] \quad (5)$$

$$RMSE = \sqrt[2]{\frac{1}{n}\sum_{i=1}^{n}(t_i - o_i)^2} \quad (6)$$

$$R^2 = \left[\frac{\left(n\sum_{i=1}^{n} t_i o_i - \left(\sum_{i=1}^{n} t_i \sum_{i=1}^{n} o_i\right)\right)^2}{\left(n\sum_{i=1}^{n} t_i^2 - \left(\sum_{i=1}^{n} t_i\right)^2\right)\left(n\sum_{i=1}^{n} o_i^2 - \left(\sum_{i=1}^{n} o_i\right)^2\right)}\right] \quad (7)$$

Where t denotes the experimental value, o refers to the predicted value, and n refers to the total number of data.

RMSE indicates the sample standard deviation between the observed and predicted values, while MAPE represents a measure of the prediction accuracy of the forecasting method in statistics, for instance, in the estimation of trends, and it generally presents accuracy as a percentage. R^2 was used as the criterion for evaluating the agreement between the experimental and predicted values. The statistical performances of the model established by GEP and the models proposed by Arora and Asef and Reddish are summarized in Table 4. As seen in Table 4, for both the training and validation sets, the GEP model had low MAPE, RMSE and high R^2 values. The R^2 values related to the experimental and predicted data from the GEP model for training and testing were 0.88 and 0.81, respectively, implying that the model was capable of explaining 88% and 0.81% of the variation in the data.

Table 4
Performance statistics of the model.

	GEP Model		Arora's Model		Asef and Reddish's Model	
	Training	Validation	Training	Validation	Training	Validation
MAPE	0.069	0.264	0.173	0.465	1.198	3.80
RMSE	1904.84	2688.59	3991.40	4085.46	27206.39	30446.23
R^2	0.88	0.81	0.56	0.60	0,62	0.62

The graphical comparison of the E values predicted by the GEP-based models and models proposed by Arora and Asef and Reddish in the training and validation phases to their experimental values is presented in Figs. 8 and 9, respectively. As is observed from the figures in question, the experimental values were close to the estimated values for the GEP-based models. They demonstrated the successful performance of GEP models in predicting E at a triaxial stress state using θ, ρ, Vp, SH, and G_1. The models proposed by Arora and Asef and Reddish exhibited a similar performance to each other in the current experimental data. However, the performances of these models are lower than that of the GEP-based model.

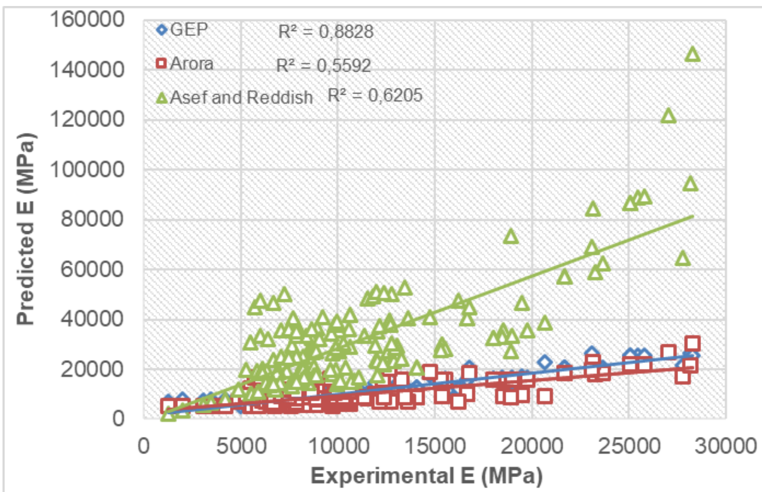


Fig. 8. Comparison of the measured and predicted E results of the training set.

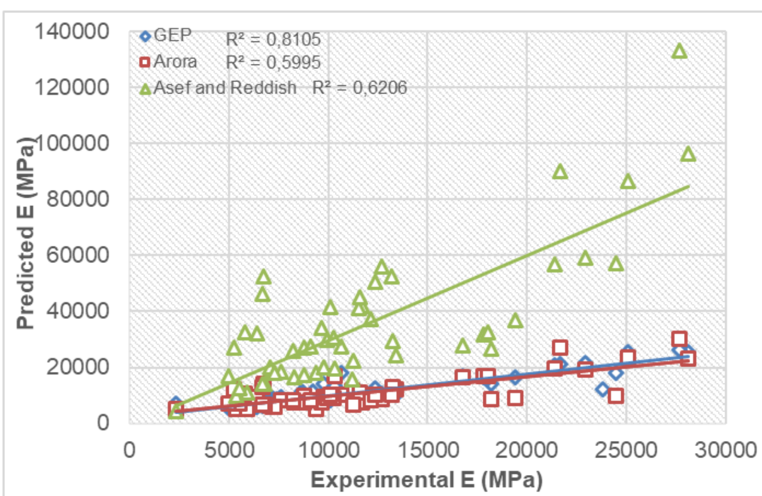


Fig. 9. Comparison of the measured and predicted E results of the validation set.

5. Conclusion

In this study, the triaxial compression test for nine different rocks (two granites, two andesites, two limestones, one tuff, one diorite, and one marble) was carried out at four confining stress values (5, 10, 15, and 20 Mpa) to determine the behavior of rocks at a triaxial stress state. The GEP-based formulation is displayed for predicting the modulus of elasticity (E) from some mass characteristics of rocks by utilizing the GEP system, which is a novel methodology for this kind of issues. In the GEP model, confining stress, density, porosity, Schmidt hardness, ultrasonic p-wave velocity, and uniaxial compressive strength were utilized as the input variables, whereas E was used as the output variable. For this purpose, 256 data sets were used. The comparison of the performance of the GEP-based model with the formulae previously suggested in the literature by Arora [19] and Asef and Reddish [20] was made using the root mean square error (RMSE) values, mean absolute percentage error (MAPE), and the coefficient of determination (R^2).

From the present study, the following was concluded:

- With the increase in the support pressure, there was a considerable increase in the modulus of elasticity. However, when the support pressure was further increased, a reduction in the increasing modulus of elasticity was determined. The reason for this may be the closure of micro and/or macro fractures under confining pressure. The mentioned finding is parallel to the findings obtained by Asef and Reddish.
- In the rocks with high Schmidt hardness values, it was observed that the increase in the modulus of elasticity was higher when the confining stress was increased. However, the increase in the modulus of elasticity was found to be lower in the rocks with low strength and high porosity.
- In the present research, a novel and practical approach to formulating the modulus of elasticity under three-axis loading conditions was applied. The suggested model is empirical, and experimental results constitute its basis. The verification of the GEP model's validity was performed by the statistical performance criteria utilized to assess the model, such as root mean square error (RMSE), mean absolute percentage error (MAPE), and the coefficient of determination (R^2). These statistical parameters of GEP formulations also contributed to the excellent performance of GEP models in predicting the modulus of elasticity for natural stones.
- The models proposed by Arora and Asef and Reddish exhibited a similar performance to each other in the current experimental data. However, the performance of these models is lower in comparison with the GEP-based model.
- GEP is an appropriate method for estimating the E value of rocks from anisotropic and heterogeneous materials as well as revealing nonlinear practical connections in which it is not possible to apply traditional techniques. Moreover, using GEP, the E value may be assessed without conducting complex and tiresome research facility tests. The proposed GEP formulations were easy since they could be used by anyone, who is not fully familiar with the GEP method, in comparison with other artificial intelligence methods.

Acknowledgments

The current study did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

Conflicts of Interest

The authors declare no conflict of interest.

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