Empirical Formulation of Ferrocement Members Moment Capacity Using Artificial Neural Networks

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ABSTRACT

In the two past decades, ferrocement members have been with a wide variety of uses in structural applications because of their unique physical properties (high surface-area-to-volume ratio and possible fabrication in any shape). In this study, two models were presented for a predict of the moment capacity of ferrocement members, one based on a back-propagation multilayer perceptron artificial neural network and the other proposing a new equation based on the multilayer perceptron network trained. These models with five input parameters including volume fraction of wire mesh, tensile strength, cube compressive strength of mortar, and width and the depth of specimens are presented. The results obtained from the two models are compared with experimental data and experimental equations such as plastic analysis, mechanism, and nonlinear regression approaches. Also, these results are compared with the results of the equations that researchers have proposed in recent years with soft computing methods (ANFIS, GEP, or GMDH). The prediction performance of the two models is significantly better than the experimental equations. These models are comparable to that of models provided with different soft computing methods to predict the moment capacity of ferrocement members. The result of this research has proposed a general equation with less mathematical complexity and more explicit.
Abbreviations

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
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<tbody>
<tr>
<td>h</td>
<td>Overall depth of the section</td>
</tr>
<tr>
<td>b</td>
<td>Width of the beam</td>
</tr>
<tr>
<td>x₁</td>
<td>Depth of the neutral axis</td>
</tr>
<tr>
<td>f&lt;sub&gt;cu&lt;/sub&gt;</td>
<td>Cube compressive strength of the matrix</td>
</tr>
<tr>
<td>f&lt;sub&gt;c&lt;/sub&gt;</td>
<td>The ultimate tensile strength of wire mesh</td>
</tr>
<tr>
<td>A&lt;sub&gt;s&lt;/sub&gt;</td>
<td>The cross-sectional area of steel</td>
</tr>
<tr>
<td>ν&lt;sub&gt;f&lt;/sub&gt;</td>
<td>Total volume of the fraction</td>
</tr>
<tr>
<td>η&lt;sub&gt;0&lt;/sub&gt;</td>
<td>Global efficiency factor of mesh reinforcement</td>
</tr>
<tr>
<td>σ&lt;sub&gt;y&lt;/sub&gt;</td>
<td>Yield tensile strength of wire mesh</td>
</tr>
<tr>
<td>f&lt;sub&gt;'c&lt;/sub&gt;</td>
<td>Cylinder compressive strength of the matrix</td>
</tr>
</tbody>
</table>

1. Introduction

In the past two decades, ferrocement has been widely used for the design of various structures including deep beams, walls, and pools. The ferrocement member consists of hydraulic cement mortar and wire mesh, which is a type of thin-shell concrete [1]. These members can be easily constructed with widely available materials in any possible shape. Therefore they can be applied for increased strength of structural members (increased flexural capacity of stone pillars, jacketing or reinforced concrete columns) [2], for improved energy absorption properties [3,4], and higher stiffness properties [2].

However, the modeling of ferrocement members is quite difficult. Initially, researchers proposed empirical equations to estimate the flexural capacity of ferrocement members but with limitations and practical applications. Mashrei et al. [5] was using 75 experimental data sets to present a model by back-propagation neural networks and adaptive neuro-fuzzy inference system (ANFIS) to estimate the moment capacity of ferrocement members. The BPNN model has two hidden layers, which have 8 and 4 neurons, respectively. In both models, 61 data are assigned to the training set and 14 data to the test set, respectively. In that study, it has been shown that the models have better results and more accuracy than the experimental equations. Based on 75 experimental data, Gandomi et al. [6] presented three models using gene expression programming (GEP) to estimate the moment capacity of ferrocement members, which yielded better results than the experimental equations. Ibrahim G. Shaaban et al. [7] have investigated the flexural behavior of 16 lightweight ferrocement beams with various types of core materials (Lightweight Concrete Core (LWC), Autoclaved Aerated lightweight brick Core (AAC), or Extruded Foam Core (EFC)) and three different mesh reinforcement. The results of this study indicated that the ferrocement beams that have the core of LWC and AAC have the highest ductility and can be a viable substitute for conventional beams in structures. Amir Hossein
Madadi et al. [8] have investigated the flexural behavior of 12 ferrocement slab panels containing expanded perlite lightweight aggregate with different physical properties such as amounts of expanded perlite and number of expanded rib lath layers) used the method of Digital image correlation (DIC). Abdussamad ISMAIL [9] presented a model to estimate the moment capacity of ferrocement members by using the self-evolving neural network model developed. Using 75 data, he has presented two different versions of the self-evolving neural network method with the names SEANN-I and SEANN-II, which have R² values for the two models test sets %90 and %91, respectively. Naderpour et al. [10] presented a model for the moment capacity of ferrocement members estimation using the Group Method of Data Handling (GMDH) method and 60 data for the training set and 15 data for the test set. Using that model, they have proposed an equation for estimating moment capacity with good accuracy.

Although in civil engineer, the models proposed by researchers with soft computing methods to predict various parameters are a common method [11–17]. But, proposed a general equation based on artificial neural networks is the relatively new method, for example, shear resistance prediction of concrete beams reinforced by FRP bars [18] and axial strength estimation of non-compact and slender square CFT columns [19]. In this study, the authors proposed a model using artificial neural networks, Backpropagation Multilayer perceptron (MLP) to estimate the moment capacity of the ferrocement members. Since artificial neural networks (ANN) look like a black box and the complex relationships created by the network between the input variables and the output parameter cannot be totally known. Therefore, the authors use the method presented in [20], which has derived a general equation based on the trained network to estimate the flexural capacity of the ferrocement members. Researchers can use the general equation and directly estimate the moment capacity using the five input parameters studied in this paper.

2. Existing models for predicting the moment capacity

In this section, models have been investigated and studied that provide a general equation to estimate the Mu of ferrocement members. The equations proposed by different researchers for estimation of the ferrocement member moment capacity were presented in Eq. (1)-(5) in Table 1, respectively. Mansur and Paramasivam [21] proposed an innovative method with the name Plastic analysis method based on the condition of equilibrium of forces. The equation presented by Paramasivam and Ravindrarajah [22] with the name Mechanism approach method is based on the plastic analysis. In this method, the neutral axis is assumed to be the highest cross-section. This method, the same as the previous method, has the drawback that the assumptions are very simplified. A non-dimensionalized regression equation was presented by Naaman and Homrich [23] to calculate the moment Mu. The equation was suggested by Gandomi et al. [6] for calculating the moment capacity of the ferrocement members based on Gene Expression Programming (GEP). Naderpour et al. [10], using the method of Group Method of Data Handling (GMDH), developed a formula to estimate the amount of flexural capacity. A typical cross-section of the ferrocement members can be seen in Fig. 1.
### Table 1
Analytical models for moment capacity of Ferrocement members.

<table>
<thead>
<tr>
<th>Models</th>
<th>Mu (Moment capacity)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plastic analysis method [21]</td>
<td>( M_u = \sigma_{tu} \times b(h - X_1) \frac{h}{2} )</td>
</tr>
<tr>
<td>Mechanism approach method</td>
<td>( M_u = \sigma_{tu} \times \frac{bh^2}{2} )</td>
</tr>
<tr>
<td>Simplified method [23]</td>
<td>( y = -0.0772x^2 + 0.422x + 0.005 )</td>
</tr>
<tr>
<td></td>
<td>( x = \frac{v_f\sigma_{xy}}{f'_c} )</td>
</tr>
<tr>
<td></td>
<td>( y = \frac{M_u}{\eta_0f'_{c}bh^2} )</td>
</tr>
<tr>
<td>GEP models [6]</td>
<td>( M_{u,GEP} = \frac{b(h-11)(h+f_{cu})}{5184} (f_{u}v_f)^{0.6} )</td>
</tr>
<tr>
<td>GMDH models [10]</td>
<td>( M_{u,GMDH} = 0.091 + \frac{0.092h_{cu}}{b} - \frac{0.042h^2}{v_f} + \frac{bv_f(1.037h^2f_{ul}-0.021v_f)}{f_{ul}} )</td>
</tr>
</tbody>
</table>

![Fig. 1. Cross section of ferrocement specimen.](image)

### 3. Model development

#### 3.1. Neural network model

Artificial neural networks (ANN) consist of a set of neurons that have the duty of connecting the layers of the network, such as the role of synapses in the human brain. Each network has three layers: input, hidden, and output. Due to the complexity of the problem, it is possible to use two or more layers in the hidden layer of the network. The neural networks are trained to process by creating a weight matrix of numerical value and bias generated by each neuron and the transfer function used in the hidden and output layers. Back-propagation was used for neural network modeling in this study [24].

#### 3.2. Database

The authors used a database compiled by Mashrei et al. [5] For the ferrocement members of nine literature [5,21,22,25–30]. They used 74 data in this study and normalization of all parameters to
numbers in the range of 0.1 to 0.9 using Eq. (6). The input and output parameters used and the statistical details of these parameters can be seen in Table 2, Table 3 and Fig. 2, respectively.

\[
X_{\text{normal}} = 0.8 \times \left( \frac{x - x_{\text{min}}}{x_{\text{max}} - x_{\text{min}}} \right) + 0.1
\]  

(6)

<table>
<thead>
<tr>
<th>Table 2</th>
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<tbody>
<tr>
<td><strong>Definition of the considered parameters.</strong></td>
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</table>

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>b (mm)</td>
<td>The width of specimens.</td>
</tr>
<tr>
<td>h (mm)</td>
<td>Total depth specimens.</td>
</tr>
<tr>
<td>( f_{\text{cu}} ) (MPa)</td>
<td>The compressive strength of ferrocement.</td>
</tr>
<tr>
<td>( f_{\text{UL}} ) (MPa)</td>
<td>The ultimate strength of wire mesh.</td>
</tr>
<tr>
<td>( \nu_f ) (%)</td>
<td>Volume fraction of wire mesh.</td>
</tr>
<tr>
<td>Mu (N.m)</td>
<td>Moment capacity of ferrocement members.</td>
</tr>
</tbody>
</table>

<table>
<thead>
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<th>Table 3</th>
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</thead>
<tbody>
<tr>
<td><strong>Database parameters range.</strong></td>
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</table>

<table>
<thead>
<tr>
<th>Variables</th>
<th>b (mm)</th>
<th>h (mm)</th>
<th>( f_{\text{cu}} ) (MPa)</th>
<th>( f_{\text{UL}} ) (MPa)</th>
<th>( \nu_f ) (%)</th>
<th>Mu (N.m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>147.432</td>
<td>42.486</td>
<td>40.317</td>
<td>545.421</td>
<td>2.441</td>
<td>757.989</td>
</tr>
<tr>
<td>Minimum</td>
<td>76</td>
<td>13</td>
<td>12.6</td>
<td>371</td>
<td>0.164</td>
<td>33</td>
</tr>
<tr>
<td>Maximum</td>
<td>400</td>
<td>100</td>
<td>62</td>
<td>979</td>
<td>8.25</td>
<td>3937</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>86.819</td>
<td>22.067</td>
<td>12.872</td>
<td>139.752</td>
<td>1.809</td>
<td>914.293</td>
</tr>
<tr>
<td>Coefficient of variation</td>
<td>58.88</td>
<td>51.94</td>
<td>31.92</td>
<td>25.62</td>
<td>74.11</td>
<td>120.62</td>
</tr>
</tbody>
</table>

Fig. 2. Parameter histograms of using the database for methods.
3.3. Proposed MLP models

The Multi-Layer Perceptron neural network trained based on 74 experimental data that assigned randomly 52 data to the training set, 7 data to the validation set, and 15 data to test set. It can be seen in Fig. 3 the structure of this network.

This network has one hidden layer and has 7 neurons that used the activation functions Tangent Sigmoid and Purelin function in the hidden and output layers, respectively. The results obtained from the network for the training, validation and testing sets can be seen in Figs. 4-7, respectively. As shown in Figs. 4-7, the correlation coefficient $R^2$ all sets are higher than 98%, and the test set error is less than 0.05%. This description, which means the trained neural network is good and accurate.

![Fig. 3. Schematic diagram of ANN models.](image)
Fig. 4. Regressions of training, validation and test data simulated by the model.

Fig. 5. Train results.
3.4. An equation for predicting moment of ferrocement members for capacity based on ANN

Given the approach proposed by Naderpour et al. [20], which presents an approach for find a relationship based on trained neural networks, after a reliable neural network model an equation for the moment capacity of ferrocement members was proposed. The most effective parameter among the input data was the width of specimens b which had the largest influence on the output parameters. The neural network was utilized to simulate the b value based on the new database.
with different values of parameter $b$ between 76mm and 400mm and the reference value for the other four parameters, presented in Table 4. Fig. 8 shows the fitting curve. The Eq. (7) obtained from the curve is as follows:

$$M'_{u} = 0.0001 \times (b)^{3} - 0.0432 \times (b)^{2} + 10.152(b) - 50.49$$  \hspace{1cm} (7)$$

Since the obtained equation cannot directly estimate the moment capacity of the ferrocement members, the correction coefficient for the other 4 parameters should be calculated and their values were given in Eq. (8).

$$Formula = M_{u} = M'_{u} \times C(h) \times C(f_{cu}) \times C(f_{ul}) \times C(\nu_{f})$$  \hspace{1cm} (8)$$

Full explanations on how to calculate correction coefficients can be found in the articles [20,31]. The correction coefficients of this study are presented for the other 4 parameters in Eq. (9)-(12). only the curves for $C(h)$ and $C(\nu_{f})$ are presented due to limitation of space and are shown in Figs 9-14.

$$C(h) = -0.4732 \left( \frac{h}{42.486} \right)^{3} + 2.3677 \left( \frac{h}{42.486} \right)^{2} - 1.2726 \left( \frac{h}{42.486} \right) + 0.3794$$  \hspace{1cm} (9)$$
\[ C(f_{cu}) = 1.2116 \left( \frac{f_{cu}}{40.317} \right)^4 - 4.427 \left( \frac{f_{cu}}{40.317} \right)^3 + 6.2701 \left( \frac{f_{cu}}{40.317} \right)^2 - 3.4063 \left( \frac{f_{cu}}{40.317} \right) + 1.3486 \] (10)

\[ C(f_{ul}) = -1.8762 \left( \frac{f_{ul}}{545.42} \right)^3 + 7.1545 \left( \frac{f_{ul}}{545.42} \right)^2 - 7.349 \left( \frac{f_{ul}}{545.42} \right) + 3.064 \] (11)

\[ C(v_{f}) = 0.0946 \left( \frac{v_f}{2.441} \right)^3 - 0.4173 \left( \frac{v_f}{2.441} \right)^2 + 1.2787 \left( \frac{v_f}{2.441} \right) + 0.0175 \] (12)

**Fig. 9.** Correction factor C(h) with various values of \( f_{cu} \).

**Fig. 10.** Correction factor C(h) with various values of \( f_{ul} \).
Fig. 11. Correction factor $C(h)$ with various values of $\nu_f$.

Fig. 12. Correction factor $C(\nu_f)$ with various values of $h$. 
4. Results and discussion

A comparison of the results of the trained Multi-Layer Perceptron neural network and the results obtained by the general equation presented in this study with the experimental data results were presented in Figs. (15)-(16). These figures show that with low and acceptable error ratios, these two models can estimate the moment capacity of ferrocement members.
Among that, model performance evaluated by RMSE, MAPE, and MAE (Eqs. (13)-(15)), was compared with the models presented in this paper with the models proposed by other researchers (Table 5). According to results presented in Table 5, the formula and the MLP presented in this study have a more effective and more appropriate evaluation performance than the experimental equations (such as plastic analysis, mechanism, and nonlinear regression approaches) proposed. The calculated error values, such as Root Mean Square Error (RMSE) of the MLP presented, have lower than the GEP method and higher than the GMDH method. The calculated coefficient of detection ($R^2$) for the MLP has 98%, which is higher than both the GEP and GMDH methods. But since this method produced the general equation (formula) from the equations obtained from the graphs, the calculated error values have slightly higher than the proposed MLP. It can be concluded that the prediction performance of the proposed models has much better than the experimental models, and the results obtained from these two models have comparable to the methods presented with soft computing. This method has proposed a general equation (presented as Eq. (8)) with less mathematical complexity and more explicit, which eases the calculation of moment capacity of the ferrocement members. Thus, researchers can be calculation $M_u$ more quickly and easily and with appropriate accuracy.

$$\text{RMSE} = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (M_u(\text{actual}) - M_u(\text{model}))}$$ (13)

$$\text{MAPE} = \frac{1}{n} \left( \frac{\sum_{i=1}^{n} |M_u(\text{actual}) - M_u(\text{model})|}{\sum_{i=1}^{n} M_u(\text{actual})} \right)$$ (14)

$$\text{MAE} = \frac{1}{n} \left( \sum_{i=1}^{n} |M_u(\text{actual}) - M_u(\text{model})| \right)$$ (15)

![Fig. 15. Verification of predicted by formula against experimental data.](image)
Table 5
Evaluation of the exiting and proposed model.

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</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>0.2167</td>
<td>0.2162</td>
<td>0.1791</td>
<td>0.2026</td>
<td>0.1444</td>
<td>0.2196</td>
<td>0.2179</td>
<td>0.2533</td>
<td>0.2491</td>
</tr>
<tr>
<td>MAE</td>
<td>0.0045</td>
<td>0.0034</td>
<td>0.0403</td>
<td>0.0388</td>
<td>0.0734</td>
<td>0.0204</td>
<td>0.0157</td>
<td>0.0174</td>
<td>0.0315</td>
</tr>
<tr>
<td>RMSE</td>
<td>0.0072</td>
<td>0.0072</td>
<td>0.0842</td>
<td>0.0774</td>
<td>0.1309</td>
<td>0.0298</td>
<td>0.0232</td>
<td>0.0255</td>
<td>0.052</td>
</tr>
<tr>
<td>MAPE</td>
<td>2.40%</td>
<td>1.70%</td>
<td>12.65%</td>
<td>12.69%</td>
<td>23.50%</td>
<td>8.88%</td>
<td>6.81%</td>
<td>7.80%</td>
<td>14.73%</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.9979</td>
<td>0.9980</td>
<td>0.8427</td>
<td>0.7637</td>
<td>0.7570</td>
<td>0.9641</td>
<td>0.9780</td>
<td>0.9835</td>
<td>0.9369</td>
</tr>
<tr>
<td>Correlation Coefficient (R)</td>
<td>0.9990</td>
<td>0.9990</td>
<td>0.9180</td>
<td>0.8739</td>
<td>0.8701</td>
<td>0.9819</td>
<td>0.9890</td>
<td>0.9917</td>
<td>0.9667</td>
</tr>
</tbody>
</table>

5. Conclusion

In this study, the authors presented two models with five input variables to predict the moment capacity of ferrocement members. One of the models for prediction of moment capacity by Artificial neural network and the other is based on neural network trained and using the process described in this article that finally to a general equation for estimation moment capacity of ferrocement members. The regression values of the chosen network trained for training and testing were 0.9961 and 0.9894 respectively and the best validation performance was 0.0010. RMSE, MAE, and MAPE were calculated as 0.0255, 0.0174, 7.80% and 0.0522, 0.0315, 14.738%, for MLP neural network trained and formula, respectively.
References


